Equation: $f(x) =$ ____________

1. domain: ____________

   range: ____________

2. $y-$intercept: ____________

   $x-$intercept(s): ____________

3. boundedness: ____________

4. horizontal asymptote(s): ____________

   vertical asymptote(s): ____________

5. increasing: ____________

   decreasing: ____________

6. absolute maximum(s): ____________

   relative maximum(s): ____________

   absolute minimum(s): ____________

   relative minimum(s): ____________

7. one-to-one: ____________

8. concave up: ____________

   concave down: ____________

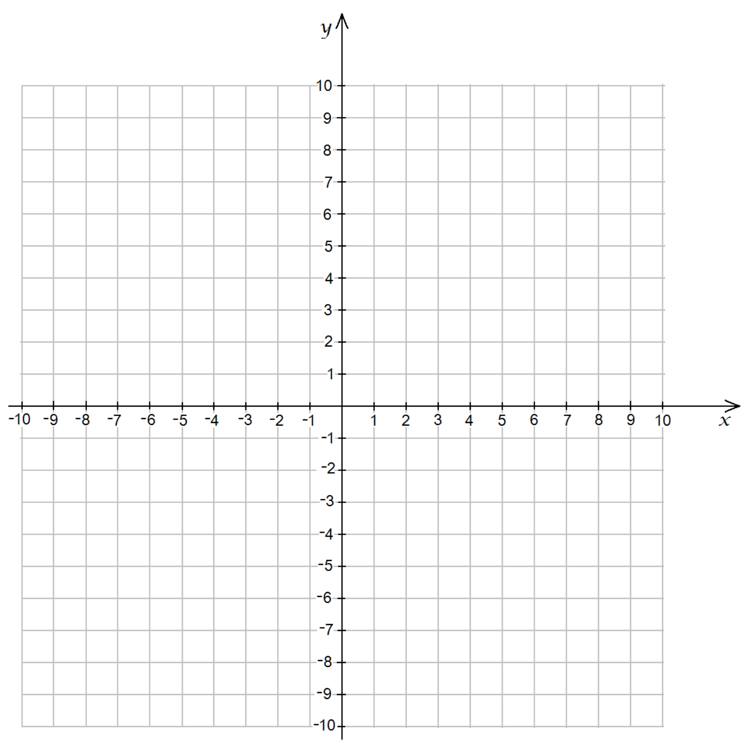
9. point(s) of inflection: ____________

10. continuous: ____________

11. even/odd: ____________

12. end-behavior:
    $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$

13. Graph



Definitions

1. The **domain** of a function is a non-empty set of elements to which we assign things.

   The **range** of a function is the set of elements that we assign to elements of the domain.

2. The $y-$**intercept** of a function is the point where the graph of the function intersects the $y-$axis. A function can have at most one $y-$intercept.

   The $x-$**intercept** of a function is the point where the graph of the function intersects the $x-$axis. A function can have several $x-$intercepts.

3. A set S is **bounded from below** if there exists a real number L (lower bound) such that for all x in S, $x \geq L$.

   A set S is **bounded from above** if there exists a real number U (upper bound) such that for all x in S, $x \leq U$.

   A set S is **bounded** if it is bounded from above and from below.

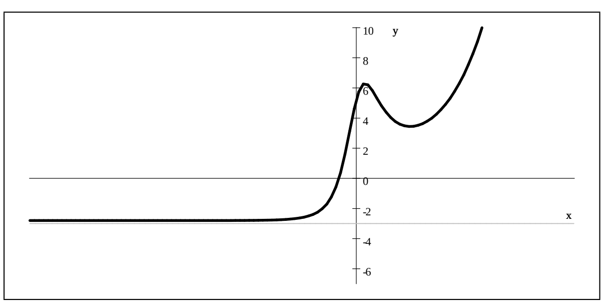
   A function f is **bounded from below** if there exists a real number L (lower bound) such that for all x in the domain, $f(x) \geq L$.

   A function f is **bounded from above** if there exists a real number U (upper bound) such that for all x in the domain, $f(x) \leq U$.

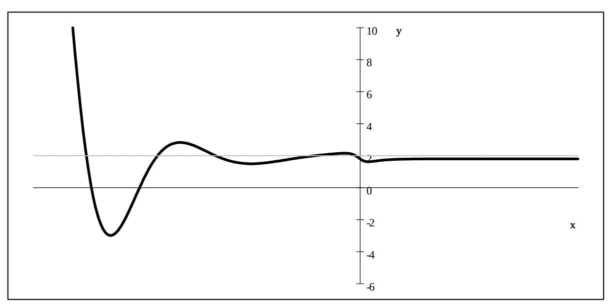
   A function f is **bounded** if it is bounded from above and from below.

4. Asymptotes. At this point we will not rigorously define asymptotes yet, just provide with an intuitive idea. There are two types of asymptotes.

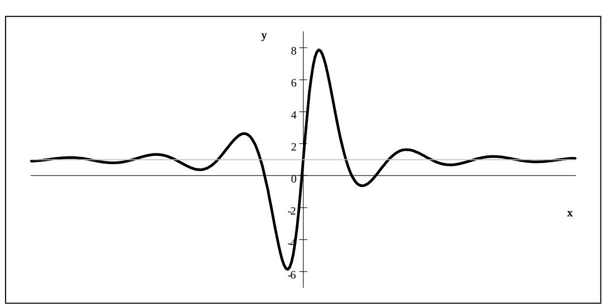
   A function can have at most two **horizontal asymptote**s. A horizontal asymptote is the graphical representation of the property that as the numbers in the domain get larger and larger (positive or negative), their assigned values get closer and closer to a fixed number.



As x gets larger and larger in the negative, $f(x)$ gets closer and closer to -3.



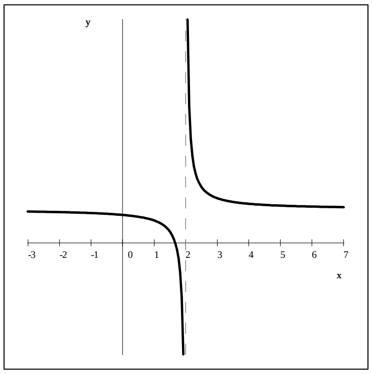
As x gets larger and larger in the positive, $f(x)$ gets closer and closer to 2.



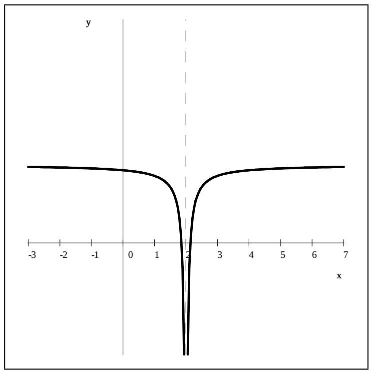
As x gets larger and larger, $f(x)$ gets closer and closer to 1.

The three examples above show different ways a function can approach a horizontal asymptote. The first picture shows a graph approaching a horizontal asympotote from above. The second graph approaches a horizontal asympotote from below. The third graph crosses it many times as it oscillates around it.

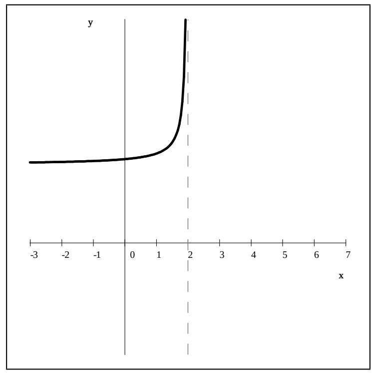
A **vertical asymptote** occurs when numbers in the domain close to a fixed number take extremely large values. A function can have many different vertical asymptotes. The function may behave in different ways at the left-hand side and at the right-hand side of the asymptote. Sometimes the function is not even defined on both sides of a vertical asymptote.



Vertical asymptote at $x = 2$



Vertical asymptote at $x = 2$



Vertical asymptote at $x = 2$

5. Definition: A function f is **increasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) \leq f(b)$.

   Definition: A function f is **strictly increasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) < f(b)$.

   Definition: A function f is **decreasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) \geq f(b)$.

   Definition: A function f is **strictly decreasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) > f(b)$.

6. Extremum is a common name for a maximum or a minimum.

   A function f has an **absolute maximum** at x_M if for all x in the domain, $f(x_M) \geq f(x)$.

   A function f has an **absolute minimum** at x_m if for all x in the domain, $f(x_m) \leq f(x)$.

   A function f has a **relative (or local) maximum** at x_M if there exists an open interval $I = (a, b)$ that contains x_M such that

   i) the function is defined on I and

   ii) if we restrict the function to I as its domain, then $(x_M, f(x_M))$ is an absolute maximum.

   A function f has a **relative (or local) minimum** at x_m if there exists an open interval $I = (a, b)$ that contains x_m such that

   i) the function is defined on I and

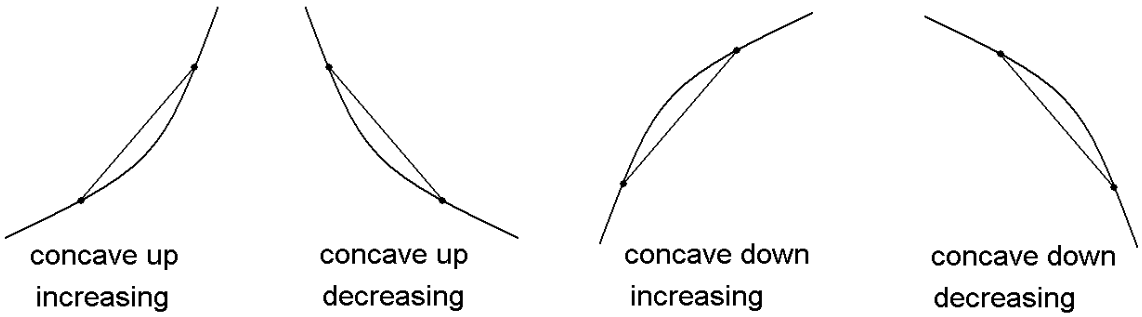
   ii) if we restrict the function to I as its domain, then $(x_m, f(x_m))$ is an absolute minimum.

7. Definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $a \neq b$, then $f(a) \neq f(b)$.

   Alternative definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $f(a) = f(b)$, then $a = b$.

8. A function f is **concave up** on an interval I if for all pairs of numbers a, b in I, the secant line connecting the points $(a, f(a))$ and $(b, f(b))$ lies above the graph of f between a and b.

   A function f is **concave down** on an interval I if for all pairs of numbers a, b in I, the secant line connecting the points $(a, f(a))$ and $(b, f(b))$ lies below the graph of f between a and b.



9. A function has **a point of inflection** at x if there exists real numbers a and b such that $a < x < b$ and f has different concavity behaviors on (a, x) and on (x, b).

10. Continuity is an extremely important concept in calculus. Definition: A function f is **continuous** at a point c if $\lim\limits_{x \to c} f(x) = f(c)$.

    Definition: A function f is **continuous** on an open interval (a, b) if for all c with $a < c < b$, f is continuous at c.

    Definition: A function f is **continuous** on a closed interval $[a, b]$ if for all c with $a < c < b$, f is continuous at c, and $\lim\limits_{x \to a^+} f(x) = f(a)$ and $\lim\limits_{x \to b^-} f(x) = f(b)$.

11. A function f is **even** if for all x in its domain, if $f(x)$ is defined, then $f(-x)$ is defined and $f(-x) = f(x)$.

    A function f is **odd** if for all x in its domain, if $f(x)$ is defined, then $f(-x)$ is defined and $f(-x) = -f(x)$.

12. The end-behavior of a function f describes the behavior of f for very large negative and very large positive numbers.

Sample Problems

Sketch the graph and give a complete analysis for each of the following functions.

1. $f(x) = \sqrt{x+1} - 2$
2. $f(x) = 10x - x^2 + 11$ where the domain is the closed interval $[3, 8]$
3. $f(x) = 10x - x^2 + 11$ where the domain is the open interval $(3, 8)$

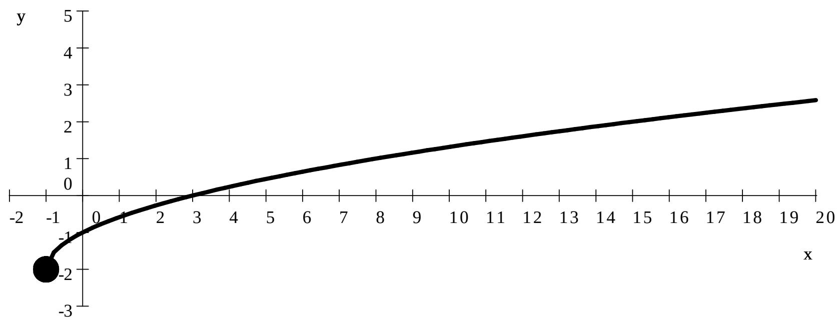
Sample Problems - Answers

1. $f(x) = \sqrt{x+1} - 2$

domain: $[-1, \infty)$
range: $[-2, \infty)$
no asymptotes
$y-$intercept: $(0, -1)$
$x-$intercept: $(3, 0)$
bounded from below

one-to-one
no relative maximum
no absolute maximum
no relative minimum
absolute minimum: $(-1, -2)$
increasing on $(-1, \infty)$

no point of inflection
concave down on $(-1, \infty)$
continuous on $(-1, \infty)$
even/odd: neither
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
$\lim_{x \to \infty} f(x) = \infty$

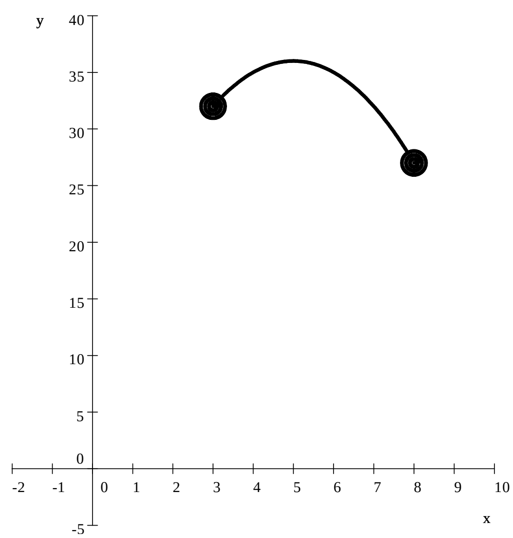


2. $f(x) = 10x - x^2 + 11$ where the domain is the closed interval $[3, 8]$.

domain: $[3, 8]$
range: $[27, 36]$
no asymptotes
no intercepts
bounded

not one-to-one
relative maximum: $(5, 36)$
absolute maximum: $(5, 36)$
no relative minimum
absolute minimum: $(8, 27)$
increasing: on $(3, 5)$
decreasing: on $(5, 8)$

no point of inflection
concave down on $[3, 8]$
continuous on $(3, 8)$
even/odd: neither
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
$\lim_{x \to \infty} f(x) =$ undefined



3. $f(x) = 10x - x^2 + 11$ where the domain is the open interval $(3, 8)$

domain: $(3, 8)$
range: $(27, 36]$
no asymptotes
no intercepts
bounded

not one-to-one
relative maximum: $(5, 36)$
absolute maximum: $(5, 36)$
no relative minimum
no absolute minimum
increasing: on $(3, 5)$
decreasing: on $(5, 8)$

no point of inflection
concave down on $(3, 8)$
continuous on $(3, 8)$
even/odd: neither
end-behavior:
$\lim_{x \to -\infty} f(x) =$ undefined
$\lim_{x \to \infty} f(x) =$ undefined