There is a significant difference between **speed** and **velocity**. Speed refers to the 'fastness' of an object's motion without any concern about its direction. For example, if car A is moving to East and car B is moving South but both cars travel exactly 45 miles in one hour, then cars A and B have the same speed, namely 45 $\frac{\text{mi}}{\text{h}}$ (miles per hour). These two cars however will have different velocities as the concept of velocity also includes the direction of movement. Two objects have the same velocity if they move with the same speed, in the same direction, along parallel lines or on the same line.

In what follows, we will consider the motion of an object that is moving along a vertical line. (Imagine an elevator in a very tall building that also have lots of underground floors.) We will describe the vertical position of the object by $L(t)$, a location function. t will denote time, measured in seconds, and L will denote the vertical position, measured in meters. So, $L(5) = -3$ means that 5 seconds after we start monitoring the object, it is 3 meters below ground level. $L(10) = 2$.

1. Suppose the location function of an object is given by $L(t) = -3t + 20$.
   a) Where is the object at the start (i.e. when we start monitoring its motion)?
   b) Where is the object 4 seconds after we start monitoring its motion?
   c) Where is the object 7 seconds after we start monitoring its motion?
2. Suppose the location function of an object is given by $L(t) = -t^2 + 4t + 8$.
   a) Where is the object at the start (i.e. when we start monitoring its motion)?
   b) Where is the object 3 seconds after we start monitoring its motion?
   c) Where is the object 7 seconds after we start monitoring its motion?

The **displacement** of an object is expressing the change in its location. If $L(t_1) = 13$ and later, $L(t_2) = 21$, then between t_1 and t_2, the object moved from a height of 13 meters to a height of 21 meters. The displacement (often denoted by s) is the change that has occured:

$$s = L(t_2) - L(t_1) = 21\,\text{m} - 13\,\text{m} = 8\,\text{m}$$

Another notation for displacement is using the capital Greek letter Δ to express change. Since displacement is the change in location, it can also be denoted by ΔL.

The displacement can easily be negative. Imagine if an object is moving downward. If $L(t_1) = 13$ and later, $L(t_2) = 2$, then the displacement is

$$s = L(t_2) - L(t_1) = 2\,\text{m} - 13\,\text{m} = -11\,\text{m}$$

A negative displacement indicates that the object has moved downward between t_1 and t_2.

3. Suppose the location function of an object is given by $L(t) = -3t + 20$.
   a) Find the displacement that occurs during the first 5 seconds.
   b) Find the displacement between $t_1 = 3\,\text{s}$ and $t_2 = 7\,\text{s}$. (s denotes seconds)
4. Suppose the location function of an object is given by $L(t) = -t^2 + 4t + 8$.
   a) Find the displacement that occurs during the first 3 seconds.
   b) Find the displacement that occurs during the first 4 seconds
   c) Find the displacement between $t_1 = 1\,\text{s}$ and $t_2 = 5\,\text{s}$.
   d) Find the displacement between $t_1 = 2\,\text{s}$ and $t_2 = 6\,\text{s}$.
   e) Find the displacement between $t_1 = 3\,\text{s}$ and $t_2 = 7\,\text{s}$.

The **average velocity** of an object is defined as the displacement divided by the time it took to travel that much.

5. Suppose the location function of an object is given by $L(t) = -3t + 20$, where t is measured in seconds and L in meters.

   a) Compute the average velocity of the object between $t_1 = 4\,\mathrm{s}$ and $t_2 = 10\,\mathrm{s}$.

   b) Compute the average velocity between $t_1 = 5\,\mathrm{s}$ and $t_2 = 8\,\mathrm{s}$.

6. Suppose the location function of an object is given by $L(t) = -t^2 + 4t + 8$, where t is measured in seconds and L in meters.

   a) Compute the average velocity between $t_1 = 0\,\mathrm{s}$ and $t_2 = 3\,\mathrm{s}$.

   b) Compute the average velocity between $t_1 = 0\,\mathrm{s}$ and $t_2 = 4\,\mathrm{s}$.

   c) Compute the average velocity between $t_1 = 5\,\mathrm{s}$ and $t_2 = 9\,\mathrm{s}$.

7. Suppose that a small object is moving up and down along a vertical line. We monitor the location of the object as a function of time. We set ground level to represent a height (or vertical location) to be zero. In each case, graph the location function given the data on the height.

   t is time, measured by seconds and h is the height, measured in meters.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
h	7	5.25	4	3.25	3	3.25	4	5.25	7	9.25	12	15.25	19	23.25	28

   a) Create a coordinate system to graph this data. Label both axis and set up a consistent scale on both of them. Then graph the data given.

   b) When is the object moving upward?

   c) What is the average velocity of the object between $t = 0$ and $t = 3$ seconds?

   d) What is the average velocity of the object between $t = 5$ seconds and $t = 10$ seconds?

Answers

1. a) $L(0) = 20$ b) $L(4) = 8$ c) $L(7) = -1$
2. a) $L(0) = 8$ b) $L(3) = 11$ c) $L(7) = -13$
3. a) $-15\,\mathrm{m}$ b) $-12\,\mathrm{m}$
4. a) 3 b) 0 c) -8 d) -16 e) -24
5. a) $-3\dfrac{\mathrm{m}}{\mathrm{s}}$ b) $-3\dfrac{\mathrm{m}}{\mathrm{s}}$
6. a) $1\dfrac{\mathrm{m}}{\mathrm{s}}$ b) $0\dfrac{\mathrm{m}}{\mathrm{s}}$ c) $-10\dfrac{\mathrm{m}}{\mathrm{s}}$
7. a) see below b) after $t = 4$ c) $-\dfrac{5}{4}\dfrac{\mathrm{ft}}{\mathrm{s}}$ d) d) $\dfrac{7}{4}\dfrac{\mathrm{ft}}{\mathrm{s}}$