Theorem: A function f is concave up if its derivative, f' is increasing. f is concave down if f' is decreasing.

It easily follows that an increasing f' means that the second derivative, f'' is positive.

$$\begin{array}{rcccl} f \text{ concave up} & \Longleftrightarrow & f' \text{ increasing} & \Longleftrightarrow & f'' \text{ positive} \\ f \text{ concave down} & \Longleftrightarrow & f' \text{ decreasing} & \Longleftrightarrow & f'' \text{ negative} \end{array}$$

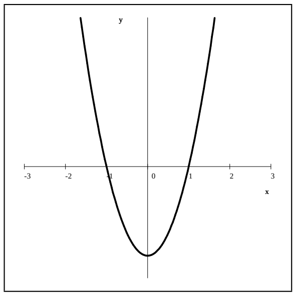
Practice Problems

In case of each of the following functions given, determine the intervals upon which the function is concave up and concave down. State the $x-$coordinate of all points of inflection.

1. $f(x) = x^4 - 6x^2 + x - 3$
2. $f(x) = x^3 + 6x^2 - 3x - 1$
3. $f(x) = x^4 - 10x^3 + 8x + 1$
4. $f(x) = 2x^4 - 8x^3 - 36x^2 - 120x + 80$
5. $f(x) = \sin x$

Answers

1. $f''(x) = 12\left(x^2 - 1\right) = 12(x+1)(x-1)$

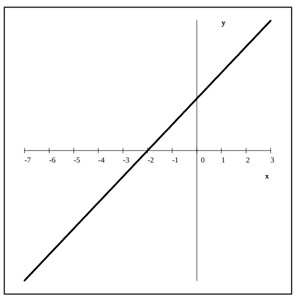


f is concave up on $(-\infty, -1)$ and on $(1, \infty)$

concave down on $(-1, 1)$

points of inflection at $x = -1$ and 1

2. $f''(x) = 6(x+2)$

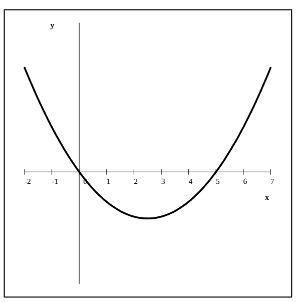


f is concave down on $(-\infty, -2)$

and concave up on $(-2, \infty)$

point of inflection at $x = -2$

3. $f''(x) = 12x(x-5)$

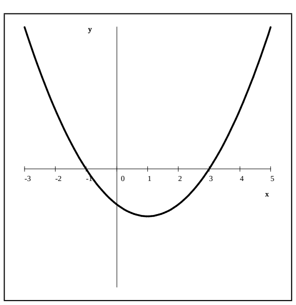


f is concave up on $(-\infty, 0)$ and on $(5, \infty)$

concave down on $(0, 5)$

points of inflection at $x = 0$ and 5

4. $f''(x) = 24x^2 - 48x - 72 = 24(x+1)(x-3)$



f is concave up on $(-\infty, -1)$ and on $(3, \infty)$

concave down on $(-1, 3)$

points of inflection at $x = -1$ and 3

5. $f''(x) = -\sin x = -f(x)$

So $\sin x$ is concave up where it is negative and concave down where it is positive. All of its zeroes are points of inflection.

Concave up: when $\pi + 2k\pi < x < 2\pi + 2k\pi$ where k is an integer

Concave down: when $2k\pi < x < \pi + 2k\pi$ where k is an integer

points of inflection: at $x = k\pi$ where k is an integer