Practice Problems

1. Differentiate each of the following. Make sure to simplify your final answer.

a) $f(x) = 5^{-2x+1}$

b) $m(y) = \log_7\left(\sqrt{y^6 + 4y^2 + 10}\right)$

c) $f(m) = \dfrac{1}{3}m^3 \ln m - \dfrac{1}{9}m^3$

d) $m(p) = \sqrt{1 - p^2}$

e) $g(t) = e^{\sin t - \cos t}$

f) $f(a) = \dfrac{2^{3a-1}}{2^{-a+1}}$

g) $L(\theta) = \dfrac{\tan\theta - \cot\theta}{\tan\theta}$

h) $v(t) = e^{2t} - e^{-2t}$

i) $f(x) = \log_2\left(x^{10}\right)$

j) $g(x) = \tan^{-1}(5x)$

k) $h(\alpha) = \log_2(\tan 2\alpha)$

l) $f(x) = \cos\left(\sin^{-1} x\right)$

m) $g(w) = \cos^{-1}\left(w^2 + 1\right)$

n) $f(y) = 2^{\log_3 y}$

o) $f(\theta) = \cot(5\theta)$

p) $g(x) = \sqrt{x + \sqrt{x}}$

q) $f(\beta) = \csc\left(\beta^2 + 1\right)$

r) $m(y) = \sec^{-1}(3y)$

s) $P(t) = \log_5(\csc t)$

2. Use implicit differentiation to find y' in each of the following.

a) $\ln\left(x^3 + y^3\right) = 5$

b) $\sin(xy) = x^2y^2$

c) $2^{x+y} = x - y$

d) $\tan^{-1} x = \cos y$

3. Compute each of the following integrals.

a) $\displaystyle\int \frac{1}{(2a-7)^4}\, da$

b) $\displaystyle\int \frac{1}{2a-7}\, da$

c) $\displaystyle\int_0^2 e^{-2t+1}\, dt$

d) $\displaystyle\int_0^1 \frac{1}{1+\theta^2}\, d\theta$

e) $\displaystyle\int \frac{1}{\sqrt{1-y^2}}\, dy$

f) $\displaystyle\int_0^3 2^x\, dx$

g) $\displaystyle\int \frac{1}{|x|\sqrt{x^2-1}}\, dx$

h) $\displaystyle\int \csc x \cot x\, dx$

i) $\displaystyle\int_0^{\pi/4} \tan^2 x\, dx$

4. Use the Fundamental Theorem of Calculus to compute each of the following derivatives.

a) $\displaystyle\frac{d}{dx}\left(\int_1^x \sqrt{t}\, dt\right)$

b) $\displaystyle\frac{d}{dx}\left(\int_x^{16} \sqrt{t}\, dt\right)$

c) $\displaystyle\frac{d}{dx}\left(\int_1^{9x} \sqrt{t}\, dt\right)$

d) $\displaystyle\frac{d}{dx}\left(\int_0^x \frac{1}{1+t^2}\, dt\right)$

e) $\displaystyle\frac{d}{dx}\left(\int_0^{x^2} \frac{1}{1+t^2}\, dt\right)$

f) $\displaystyle\frac{d}{dx}\left[\left(\int_0^x \frac{1}{1+t^2}\, dt\right)^2\right]$

Practice Problems - Answers

1.) a) $f'(x) = -2(\ln 5)\, 5^{-2x+1}$ b) $m'(y) = \dfrac{3y^5 + 4y}{\ln 7\,(y^6 + 4y^2 + 10)}$ c) $f'(m) = m^2 \ln m$

d) $m'(p) = -\dfrac{p}{\sqrt{1-p^2}}$ e) $g'(t) = (\cos t + \sin t)\, e^{\sin t - \cos t}$ f) $f'(a) = (\ln 2)\, 2^{4a}$

g) $L'(\theta) = \dfrac{2\cos\theta}{\sin^3\theta} = 2\cot^3\theta + 2\cot\theta$ h) $v'(t) = 2e^{2t} + 2e^{-2t}$ i) $f'(x) = \dfrac{10}{x\ln 2}$

j) $g'(x) = \dfrac{5}{25x^2+1}$ k) $h'(\alpha) = \dfrac{2}{\ln 2}(\cot 2\alpha + \tan 2\alpha) = \dfrac{2\tan^2 2\alpha + 2}{(\ln 2)\tan 2\alpha}$ l) $f'(x) = -\dfrac{x}{\sqrt{1-x^2}}$

m) $g'(w) = \dfrac{-2w}{\sqrt{1-(w^2+1)^2}}$ n) $f'(y) = \dfrac{\ln 2}{y\ln 3} 2^{\log_3 y}$ o) $f'(\theta) = -5\csc^2 5\theta = -5\cot^2 5\theta - 5$

p) $g'(x) = \dfrac{1}{2\sqrt{x+\sqrt{x}}}\left(\dfrac{1}{2\sqrt{x}} + 1\right)$ q) $f'(\beta) = -2\beta\dfrac{\cos(\beta^2+1)}{\sin^2(\beta^2+1)} = -2\beta\csc(\beta^2+1)\cot(\beta^2+1)$

r) $m'(y) = \dfrac{1}{|y|\sqrt{9y^2-1}}$ s) $P'(t) = -\dfrac{\cot t}{\ln 5}$

2.) a) $y' = -\dfrac{x^2}{y^2}$ b) $y' = \dfrac{-y\cos xy + 2xy^2}{x\cos xy - 2x^2 y}$ c) $y' = \dfrac{-(\ln 2)\, 2^{x+y} + 1}{(\ln 2)\, 2^{x+y} + 1}$ d) $y' = -\dfrac{1}{(x^2+1)\sin y}$

3.) a) $\dfrac{-1}{6(2a-7)^3} + C$ b) $\dfrac{1}{2}\ln|2a-7| + C$ c) $\dfrac{1}{2}e - \dfrac{1}{2}e^{-3}$ d) $\dfrac{\pi}{4}$ e) $\sin^{-1} y + C$ f) $\dfrac{7}{\ln 2}$

g) $\sec^{-1} x + C$ h) $-\csc x + C$ i) $1 - \dfrac{\pi}{4}$

4.) a) $\sqrt{x}$ b) $-\sqrt{x}$ c) $27\sqrt{x}$ d) $\dfrac{1}{1+x^2}$ e) $\dfrac{2x}{1+x^4}$ f) $\dfrac{2\tan^{-1} x}{1+x^2}$

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