

Sample Problems

Differentiate each of the following functions.

1. $f(x) = -(5x + 1)^{12}$

4. $f(x) = \frac{(x + 3)^4 - (x - 3)^4}{2}$

7. $f(x) = (x - 1)^{10} (4 - x)^5$

2. $f(x) = \sqrt{x^4 - 1}$

5. $f(x) = \log_x(x^2 + 1)$

8. $f(x) = \frac{5x - 1}{5x + 1}$

3. $f(x) = \frac{(x + 3)^4 + (x - 3)^4}{2}$

6. $f(x) = \ln(\ln(x^5))$

Practice Problems

Differentiate each of the following functions. Assume that $a, b,$ and c are constants.

1. $f(x) = \log_7(x^2 - 1)$

11. $f(x) = \frac{3 - 2x}{3 + 2x}$

2. $f(x) = \frac{x^5 - 3x^2 + 19}{2x}$

12. $f(x) = (\ln(3x) - \ln(5x))^4$

3. $f(x) = \frac{x^2}{2x + 1}$

13. $f(x) = \frac{x^2 + 6x + 5}{x + 1}$

4. $f(x) = xe^{-5x}$

14. $f(x) = 2^{3x} \cdot \log_5 2x$

5. $f(x) = \frac{x + 7}{x - 7}$

15. $f(x) = \left(\frac{1}{x} - \frac{1}{x^2}\right)(2x^3 + 4)$

6. $f(x) = \sqrt[3]{5x^2 - 7x + 11^x - e^8}$

16. $f(x) = \frac{1}{\ln(x^7 - 9x^2 + 1)}$

7. $f(x) = -\frac{2}{3}(x^7 + 2x - 15)$

17. $f(x) = \frac{ax + b}{bx + c}$

8. $f(x) = \frac{a}{b + \sqrt{x}}$

18. $f(x) = x \ln x - x$

9. $f(x) = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$

19. $f(x) = \frac{8x^2 - 20x + 1}{4x - 10}$

10. $f(x) = \frac{e^x - e^{-x}}{2}$

20. $f(x) = \frac{2}{3}(x + 1)\sqrt{x + 1}$

Sample Problems - Answers

1. $f'(x) = -60(5x + 1)^{11}$
2. $f'(x) = \frac{2x^3}{\sqrt{x^4 - 1}}$
3. $f'(x) = 4x^3 + 108x$
4. $f'(x) = 36x^2 + 108$
5. $f'(x) = \frac{\frac{2x \ln x}{x^2 + 1} - \frac{\ln(x^2 + 1)}{x}}{(\ln x)^2}$
6. $f'(x) = \frac{1}{x \ln x}$
7. $f'(x) = -15(x - 1)^9(x - 3)(x - 4)^4$
8. $f'(x) = \frac{10}{(5x + 1)^2}$

Practice Problems - Answers

1. $f'(x) = \frac{2x}{(x^2 - 1) \ln 7}$
2. $f'(x) = 2x^3 - \frac{19}{2x^2} - \frac{3}{2}$
3. $f'(x) = \frac{2x^2 + 2x}{(2x + 1)^2}$
4. $f'(x) = e^{-5x} - 5xe^{-5x}$
5. $f'(x) = -\frac{14}{(x - 7)^2}$
6. $f'(x) = \frac{10x + (\ln 11) 11^x - 7}{3(5x^2 - 7x + 11^x - e^8)^{2/3}}$
7. $f'(x) = -\frac{14}{3}x^6 - \frac{4}{3}$
8. $f'(x) = -\frac{a}{2\sqrt{x}(b + \sqrt{x})^2}$
9. $f'(x) = xe^{4x}$
10. $f'(x) = \frac{e^x + e^{-x}}{2}$
11. $f'(x) = -\frac{12}{(2x + 3)^2}$
12. $f'(x) = 0$
13. $f'(x) = 1$
14. $f'(x) = 3\frac{\ln 2}{\ln 5}(\ln x + \ln 2)2^{3x} + \frac{2^{3x}}{x \ln 5}$
15. $f'(x) = 4x - \frac{4}{x^2} + \frac{8}{x^3} - 2$
16. $f'(x) = -\frac{7x^6 - 18x}{(x^7 - 9x^2 + 1) \ln^2(x^7 - 9x^2 + 1)}$
17. $f'(x) = \frac{ac - b^2}{(bx + c)^2}$
18. $f'(x) = \ln x$
19. $f'(x) = \frac{8x^2 - 40x + 49}{4x^2 - 20x + 25} = 2 - \frac{1}{(2x - 5)^2}$
20. $f'(x) = \sqrt{x + 1}$

Sample Problems - Solutions

Differentiate each of the following functions.

$$1. f(x) = -(5x + 1)^{12} \quad f'(x) = -60(5x + 1)^{11}$$

Solution: We apply the chain rule.

$$f'(x) = -12 \cdot (5x + 1)^{11} \cdot (5) = -60(5x + 1)^{11}$$

$$2. f(x) = \sqrt{x^4 - 1} \quad f'(x) = \frac{2x^3}{\sqrt{x^4 - 1}}$$

Solution: We re-write f with exponential notation and differentiate it via the chain rule.

$$\begin{aligned} f(x) &= (x^4 - 1)^{1/2} \\ f'(x) &= \frac{1}{2}(x^4 - 1)^{-1/2} \cdot (4x^3) = \frac{2x^3}{\sqrt{x^4 - 1}} \end{aligned}$$

$$3. f(x) = \frac{(x + 3)^4 + (x - 3)^4}{2} \quad f'(x) = 4x^3 + 108x$$

Solution: We re-write f and then apply the chain rule.

$$\begin{aligned} f(x) &= \frac{(x + 3)^4 + (x - 3)^4}{2} = \frac{1}{2}(x + 3)^4 + \frac{1}{2}(x - 3)^4 \\ f'(x) &= \frac{1}{2}(4)(x + 3)^3(1) + \frac{1}{2}(4)(x - 3)^3(1) = 2(x + 3)^3 + 2(x - 3)^3 \end{aligned}$$

Thus $f'(x) = 2(x + 3)^3 + 2(x - 3)^3$ or, as a polynomial,

$$\begin{aligned} f'(x) &= 2(x + 3)^3 + 2(x - 3)^3 = 2(x^3 + 9x^2 + 27x + 27 + x^3 - 9x^2 + 27x - 27) \\ &= 2(2x^3 + 54x) = 4x^3 + 108x \end{aligned}$$

$$4. f(x) = \frac{(x + 3)^4 - (x - 3)^4}{2} \quad f'(x) = 36x^2 + 108$$

Solution: We re-write f and then apply the chain rule.

$$\begin{aligned} f(x) &= \frac{(x + 3)^4 - (x - 3)^4}{2} = \frac{1}{2}(x + 3)^4 - \frac{1}{2}(x - 3)^4 \\ f'(x) &= \frac{1}{2}(4)(x + 3)^3(1) - \frac{1}{2}(4)(x - 3)^3(1) = 2(x + 3)^3 - 2(x - 3)^3 \end{aligned}$$

Thus $f'(x) = 2(x + 3)^3 - 2(x - 3)^3$ or, as a polynomial,

$$\begin{aligned} f'(x) &= 2(x + 3)^3 - 2(x - 3)^3 = 2(x^3 + 9x^2 + 27x + 27 - (x^3 - 9x^2 + 27x - 27)) \\ &= 2(x^3 + 9x^2 + 27x + 27 - x^3 + 9x^2 - 27x + 27) \\ &= 2(18x^2 + 54) = 36x^2 + 108 \end{aligned}$$

$$5. f(x) = \log_x(x^2 + 1) \quad f'(x) = \frac{\frac{2x \ln x}{x^2 + 1} - \frac{\ln(x^2 + 1)}{x}}{(\ln x)^2}$$

Solution: We re-write f and apply the quotient rule. To differentiate $\ln(x^2 + 1)$, we apply the chain rule.

$$\begin{aligned} f'(x) &= (\log_x(x^2 + 1))' = \left(\frac{\ln(x^2 + 1)}{\ln x} \right)' = \frac{(\ln(x^2 + 1))' \ln x - \ln(x^2 + 1) (\ln x)'}{(\ln x)^2} \\ &= \frac{\left(\frac{1}{x^2 + 1} (2x) \right)' \ln x - \ln(x^2 + 1) \left(\frac{1}{x} \right)}{(\ln x)^2} = \frac{\frac{2x \ln x}{x^2 + 1} - \frac{\ln(x^2 + 1)}{x}}{(\ln x)^2} \end{aligned}$$

$$6. f(x) = \ln(\ln(x^5)) \quad f'(x) = \frac{1}{x \ln x}$$

Solution: We re-write f using properties of logarithms and apply the chain rule.

$$\begin{aligned} f(x) &= \ln(\ln(x^5)) = \ln(5 \ln x) = \ln 5 + \ln \ln x \\ f'(x) &= (\ln 5 + \ln \ln x)' = 0 + \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x} \end{aligned}$$

$$7. f(x) = (x - 1)^{10} (4 - x)^5 \quad f'(x) = -15(x - 1)^9 (x - 3)(x - 4)^4$$

Solution: We first factor out the leading coefficients. Then we differentiate f using the product rule.

$$\begin{aligned} f(x) &= (x - 1)^{10} (4 - x)^5 = -(x - 1)^{10} (x - 4)^5 \\ f'(x) &= -(10(x - 1)^9 (x - 4)^5 + (x - 1)^{10} (5)(x - 4)^4) = -(10(x - 1)^9 (x - 4)^5 + 5(x - 1)^{10} (x - 4)^4) \end{aligned}$$

From this expression, we can factor out $5(x - 1)^9 (x - 4)^4$

$$\begin{aligned} f'(x) &= -5(x - 1)^9 (x - 4)^4 (2(x - 4) + (x - 1)) = -5(x - 1)^9 (x - 4)^4 (3x - 9) \\ &= -5(x - 1)^9 (x - 4)^4 (3)(x - 3) = -15(x - 1)^9 (x - 3)(x - 4)^4 \end{aligned}$$

$$8. f(x) = \frac{5x - 1}{5x + 1} \quad f'(x) = \frac{10}{(5x + 1)^2}$$

Solution: Method 1

Although you may use the quotient rule, it can be avoided with a little algebraic transformation before the calculus:

$$\begin{aligned} f(x) &= \frac{5x - 1}{5x + 1} = \frac{5x + 1 - 1 - 1}{5x + 1} = \frac{5x + 1}{5x + 1} - \frac{2}{5x + 1} = 1 - \frac{2}{5x + 1} = 1 - 2(5x + 1)^{-1} \\ f'(x) &= (1 - 2(5x + 1)^{-1})' = 0 - 2(-1)(5x + 1)^{-2} (5) = 10(5x + 1)^{-2} = \frac{10}{(5x + 1)^2} \end{aligned}$$

Method 2

Apply the quotient rule.

$$\begin{aligned} f'(x) &= \left(\frac{5x - 1}{5x + 1} \right)' = \frac{(5x - 1)'(5x + 1) - (5x - 1)(5x + 1)'}{(5x + 1)^2} \\ &= \frac{5(5x + 1) - 5(5x - 1)}{(5x + 1)^2} = \frac{25x + 5 - 25x + 5}{(5x + 1)^2} = \frac{10}{(5x + 1)^2} \end{aligned}$$

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