Practice Problems

Differentiate each of the following functions. (Please note that in many expercises, there is a way to avoid using the quotient rule.)

1. $f(x) = e^{x^2-3x+5}$
2. $f(x) = \dfrac{x^2-5x+6}{x-3}$
3. $f(x) = 2^{\sin x}$
4. $f(x) = \log_{10}\left(x^4+8x^2+e\right)$
5. $f(x) = \sin(5^x)$
6. $f(x) = e^{-x}$
7. $f(x) = \dfrac{1}{2}(e^x - e^{-x})$
8. $f(x) = \dfrac{1}{2}xe^{2x} - \dfrac{1}{4}e^{2x}$
9. $f(x) = x^3 + 3^x$
10. $f(x) = \dfrac{e^x}{e^x+1}$
11. $f(x) = \dfrac{x-2}{x+5}$
12. $f(x) = 2^{\ln x}$
13. $f(x) = \tan x$
14. $f(x) = \dfrac{x}{x-1}$
15. $f(x) = \ln(\tan x)$
16. $f(x) = \ln\left(\dfrac{3-\sin x}{3+\sin x}\right)^4$
17. $f(x) = \cos\left(5^{2x-3}\right)$
18. $f(x) = \dfrac{2^{5x-1}}{2^{2x+1}}$
19. $f(x) = \dfrac{x^8}{\ln x}$
20. $f(x) = e^{\sin x + \cos x}$
21. $f(x) = \tan(e^{-x})$
22. $f(x) = \dfrac{2^x+1}{2^x-1}$

Define $f(x) = \dfrac{1}{2}(e^x + e^{-x})$ and $g(x) = \dfrac{1}{2}(e^x - e^{-x})$. These functions have very interesting properties.

23. Compute $f'(x)$.
24. Compute $g'(x)$.
25. Compute $f(0)$ and $g(0)$.
26. Compute $(f(x))^2 - (g(x))^2$.

As we have already seen, these functions have properties similar to trigonometric functions. In fact, $f(x)$ is really called $\cosh x$ (hyperbolic cosine of x) and $g(x)$ is called $\sinh x$ (hyperbolic sine of x)

$$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right) \quad \text{and} \quad \sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$$

There are more properties that make them so similar to trignometric functions.

27. Prove that $\cosh x$ is an even function and $\sinh x$ is an odd function.
28. Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$
29. Prove that $2\sinh x \cosh x = \sinh 2x$

Practice Problems - Answers

1.) $f'(x) = (2x-3)\,e^{x^2-3x+5}$ 2.) $f'(x) = 1$ 3.) $f'(x) = (\ln 2)(\cos x)\,2^{\sin x}$

4.) $f'(x) = \dfrac{4x^3+16x}{(x^4+8x^2+e)\ln 10}$ 5.) $f'(x) = (\ln 5)(5^x)\cos(5^x)$ 6.) $f'(x) = -e^{-x}$

7.) $f'(x) = \dfrac{1}{2}(e^x + e^{-x})$ 8.) $f'(x) = xe^{2x}$ 9.) $f'(x) = 3x^2 + (\ln 3)\,3^x$ 10.) $f'(x) = \dfrac{e^x}{(e^x+1)^2}$

11.) $f'(x) = \dfrac{7}{(x+5)^2}$ 12.) $f'(x) = \left(\dfrac{\ln 2}{x}\right)2^{\ln x}$ 13.) $f'(x) = \tan^2 x + 1 = \sec^2 x$

14.) $f'(x) = -\dfrac{1}{(x-1)^2}$ 15.) $f'(x) = \dfrac{\sec^2 x}{\tan x} = \dfrac{\tan^2 x + 1}{\tan x} = \tan x + \dfrac{1}{\tan x} = \tan x + \cot x$

16.) Note: it is easier if we re-write f as $4\ln(3-\sin x) - 4\ln(3+\sin x)$

$$f'(x) = -4\cos x\left(\frac{1}{\sin x + 3} + \frac{1}{-\sin x + 3}\right)$$

17.) $f'(x) = -2(\ln 5)\left(5^{2x-3}\right)\sin\left(5^{2x-3}\right)$ 18.) $f'(x) = 3\ln 2\cdot 2^{3x-2}$ 19.) $f'(x) = \dfrac{x^7(8\ln x - 1)}{\ln^2 x}$

20.) $f'(x) = e^{\cos x + \sin x}(\cos x - \sin x)$ 21.) $f'(x) = -e^{-x}\left(\tan^2(e^{-x}) + 1\right)$ 22.) $f'(x) = \dfrac{-2(\ln 2)\,2^x}{(2^x-1)^2}$

23.) $f'(x) = \dfrac{1}{2}(e^x - e^{-x}) = g(x)$ 24.) $g'(x) = \dfrac{1}{2}(e^x + e^{-x}) = f(x)$ 25.) $f(0) = 1$ and $g(0) = 0$

26.) 1 27.)

$$\cosh(-x) = \frac{1}{2}\left(e^{-x} + e^{-(-x)}\right) = \cosh x and \sinh(-x) = \frac{1}{2}\left(e^{-x} - e^{-(-x)}\right) = \frac{1}{2}\left(e^{-x} - e^x\right) = -\frac{1}{2}\left(e^x - e^{-x}\right) = -\sinh x$$

28.)

$$\begin{aligned}
\cosh^2 x + \sinh^2 x &= \left[\frac{1}{2}\left(e^x + e^{-x}\right)\right]^2 + \left[\frac{1}{2}\left(e^x - e^{-x}\right)\right]^2 \\
&= \frac{1}{4}\left((e^x)^2 + \left(e^{-x}\right)^2 + 2e^x\left(e^{-x}\right)\right) + \frac{1}{4}\left((e^x)^2 + \left(e^{-x}\right)^2 - 2e^x\left(e^{-x}\right)\right) \\
&= \frac{1}{4}\left[e^{2x} + e^{-2x} + 2 + e^{2x} + e^{-2x} - 2\right] = \frac{1}{4}\left(2e^{2x} + 2e^{-2x}\right) = \frac{1}{2}\left(e^{2x} + e^{-2x}\right) = \cosh 2x
\end{aligned}$$

29.)

$$\begin{aligned}
2\sinh x\cosh x &= 2\left[\frac{1}{2}\left(e^x - e^{-x}\right)\right]\left[\frac{1}{2}\left(e^x + e^{-x}\right)\right] = 2\cdot\frac{1}{4}\left[\left(e^x - e^{-x}\right)\left(e^x + e^{-x}\right)\right] = \frac{1}{2}\left[(e^x)^2 - \left(e^{-x}\right)^2\right] \\
&= \frac{1}{2}\left[e^{2x} - e^{-2x}\right] = \sinh 2x
\end{aligned}$$

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