

Differentiate each of the following functions.

1. $f(x) = \frac{\sin x}{x}$

5. $f(x) = \frac{1}{x^2 + 1}$

9. $f(x) = \frac{\cos x}{\log_3 x}$

2. $f(x) = \frac{x^4 - x^2 + 1}{\cos x}$

6. $f(x) = \frac{\ln x}{x}$

10. $f(x) = \frac{\sqrt{x}}{x^3}$

3. $f(x) = \frac{x+1}{x-1}$

7. $f(\theta) = \tan \theta$

11. $f(x) = \log_2 x + \log_x 2$

4. $f(x) = \frac{x^3 - 1}{x^2}$

8. $f(\theta) = \sec \theta$

12. $f(x) = \frac{1 + \ln x}{x^2 - \ln x}$

13. Preview of calculus 2. A bit more on tangent and secant.

a) Prove the identity $\tan^2 x + 1 = \sec^2 x$

b) Based on the identity above, re-write $f'(x)$ when $f(x) = \tan x$.

c) Compute $f'(x)$ if $f(x) = \sec x$ and re-write it in terms of tangent and/or secant.

d) Repeat this with co-tangent and co-secant.

Answers

$$1. f'(x) = \frac{x \cos x - \sin x}{x^2} \quad 2. f'(x) = \frac{(4x^3 - 2x) \cos x + (x^4 - x^2 + 1) \sin x}{\cos^2 x} \quad 3. f'(x) = -\frac{2}{(x-1)^2}$$

$$4. f'(x) = 1 + \frac{2}{x^3} \text{ or } \frac{x^3 + 2}{x^3} \quad 5. f'(x) = \frac{-2x}{(x^2 + 1)^2} \quad 6. f'(x) = \frac{1 - \ln x}{x^2} \quad 7. f'(\theta) = \frac{1}{\cos^2 \theta}$$

$$8. f'(\theta) = \frac{\sin \theta}{\cos^2 \theta} \quad \text{Note that this result can also be written as } \frac{\sin \theta}{\cos^2 \theta} = \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} = \tan \theta \sec \theta$$

$$9. f'(x) = \frac{-\sin x \log_3 x - \cos x \frac{1}{x \ln 3}}{(\log_3 x)^2} = \frac{-x \ln 3 \sin x \log_3 x - \cos x}{(\log_3 x)^2 x \ln 3} = \frac{-x \sin x \ln x - \cos x}{(\log_3 x)^2 x \ln 3}$$

$$= \frac{-x \sin x \ln x - \cos x}{\left(\frac{\ln x}{\ln 3}\right)^2 x \ln 3} = \frac{-\ln 3 (x \ln x \sin x + \cos x)}{x (\ln x)^2}$$

$$10. f'(x) = -\frac{5\sqrt{x}}{2x^4} \quad 11. f'(x) = \frac{1}{x \ln 2} - \frac{\ln 2}{x \ln^2 x} \quad 12. f'(x) = \frac{-x^2 - 2x^2 \ln x + 1}{x(x^2 - \ln x)^2}$$

$$13. \text{ a) } \tan^2 x + 1 = \sec^2 x$$

$$\text{LHS} = \tan^2 x + 1 = \frac{\sin^2 x}{\cos^2 x} + 1 = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

$$\text{b) } f'(x) = \frac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x \quad \text{c) } f'(x) = \sec x \tan x \quad \text{d) } (\cot x)' = -\csc^2 x = -1 - \cot^2 x$$

$$\text{e) } (\csc x)' = -\csc x \cot x$$