

## Sample Problems

Evaluate each of the following integrals.

1.)  $\int_1^{\infty} \frac{1}{x^4} dx$

4.)  $\int_0^{\infty} e^{-5x} dx$

7.)  $\int_2^{\infty} \frac{1}{\sqrt[3]{2x-1}} dx$

2.)  $\int_1^{\infty} \frac{1}{x} dx$

5.)  $\int_0^{\infty} xe^{-x^2} dx$

3.)  $\int_{10}^{\infty} \frac{1}{x \ln x} dx$

6.)  $\int_2^{\infty} \frac{x^2}{(x^3-1)^4} dx$

## Practice Problems

Evaluate each of the following integrals.

1.)  $\int_1^{\infty} \frac{1}{x^2} dx$

4.)  $\int_0^{\infty} x^2 e^{-x^3+1} dx$

7.)  $\int_0^{\infty} \frac{1}{\sqrt{3x+1}} dx$

2.)  $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$

5.)  $\int_0^{\infty} \frac{1}{2} e^{-x/2} dx$

8.)  $\int_0^{\infty} \frac{1}{(3x+1)^4} dx$

3.)  $\int_4^{\infty} \frac{1}{x (\ln x)^3} dx$

6.)  $\int_1^{\infty} \frac{1}{\sqrt{x} (\sqrt{x}+1)^3} dx$

9.)  $\int_0^{\infty} \frac{1}{2^{x-1}} dx$

## Sample Problems - Answers

$$1.) \frac{1}{3} \quad 2.) \infty \quad 3.) \infty \quad 4.) \frac{1}{5} \quad 5.) \frac{1}{2} \quad 6.) \frac{1}{3087} \quad 7.) \infty$$

## Practice Problems - Answers

$$1.) 1 \quad 2.) \infty \quad 3.) \frac{1}{2 \ln^2 4} \quad 4.) \frac{1}{3} e \quad 5.) 1 \quad 6.) \frac{1}{4}$$

$$7.) \infty \quad 8.) \frac{1}{9} \quad 9.) \frac{2}{\ln 2}$$

## Sample Problems - Solutions

$$1. \int_1^{\infty} \frac{1}{x^4} dx$$

$$\int_1^{\infty} \frac{1}{x^4} dx = \lim_{N \rightarrow \infty} \int_1^N x^{-4} dx = \lim_{N \rightarrow \infty} \left. \frac{x^{-3}}{-3} \right|_1^N = \lim_{N \rightarrow \infty} \left( \frac{N^{-3}}{-3} - \frac{1^{-3}}{-3} \right) = \lim_{N \rightarrow \infty} \left( \frac{-1}{3N^3} - \left( \frac{-1}{3} \right) \right) = \frac{1}{3}$$

$$2. \int_1^{\infty} \frac{1}{x} dx$$

$$\int_1^{\infty} \frac{1}{x} dx = \lim_{N \rightarrow \infty} \int_1^N \frac{1}{x} dx = \lim_{N \rightarrow \infty} \ln |x| \Big|_1^N = \lim_{N \rightarrow \infty} (\ln N - \ln 1) = \infty$$

$$3. \int_{10}^{\infty} \frac{1}{x \ln x} dx$$

We first compute the indefinite integral, using substitution. Let  $u = \ln x$ . Then  $du = \frac{1}{x} dx$  and so  $dx = x du$ .

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{xu} x du = \int \frac{1}{u} du = \ln |u| + C = \ln |\ln x| + C$$

Now we are ready to evaluate the improper integral.

$$\int_{10}^{\infty} \frac{1}{x \ln x} dx = \lim_{N \rightarrow \infty} \int_{10}^N \frac{1}{x \ln x} dx = \lim_{N \rightarrow \infty} \ln \ln |x| \Big|_{10}^N = \lim_{N \rightarrow \infty} (\ln(\ln N) - \ln(\ln 10)) = \infty$$

$$4. \int_0^{\infty} e^{-5x} dx$$

$$\int_0^{\infty} e^{-5x} dx = \lim_{N \rightarrow \infty} \int_0^N e^{-5x} dx = \lim_{N \rightarrow \infty} \left. \frac{e^{-5x}}{-5} \right|_0^N = \lim_{N \rightarrow \infty} \left( \frac{e^{-5N}}{-5} - \frac{e^{-5(0)}}{-5} \right) = \lim_{N \rightarrow \infty} \left( \frac{-1}{-5e^{5N}} - \frac{1}{-5} \right) = \frac{1}{5}$$

$$5. \int_0^{\infty} x e^{-x^2} dx$$

We first compute the indefinite integral, using substitution. Let  $u = -x^2$ . Then  $du = -2x dx$  and so  $dx = \frac{du}{-2x}$ .

$$\int x e^{-x^2} dx = \int x e^u \frac{du}{-2x} = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{-x^2} + C$$

Now we are ready to evaluate the improper integral.

$$\int_0^{\infty} x e^{-x^2} dx = \lim_{N \rightarrow \infty} \int_0^N x e^{-x^2} dx = \lim_{N \rightarrow \infty} \left. -\frac{1}{2} e^{-x^2} \right|_0^N = -\frac{1}{2} \lim_{N \rightarrow \infty} \left. \frac{1}{e^{x^2}} \right|_0^N = -\frac{1}{2} \lim_{N \rightarrow \infty} \left( \frac{1}{e^{N^2}} - \frac{1}{e^{0^2}} \right) = \frac{1}{2}$$

$$6. \int_2^{\infty} \frac{x^2}{(x^3 - 1)^4} dx$$

We first compute the indefinite integral, using substitution. Let  $u = x^3 - 1$ . Then  $du = 3x^2 dx$  and so  $dx = \frac{du}{3x^2}$ .

$$\int \frac{x^2}{(x^3 - 1)^4} dx = \int \frac{x^2}{u^4} \frac{du}{3x^2} = \frac{1}{3} \int u^{-4} du = \frac{1}{3} \cdot \frac{u^{-3}}{-3} + C = -\frac{1}{9u^3} + C = \frac{-1}{9(x^3 - 1)^3} + C$$

Now we are ready to evaluate the improper integral.

$$\begin{aligned} \int_2^{\infty} \frac{x^2}{(x^3 - 1)^4} dx &= \lim_{N \rightarrow \infty} \int_2^N \frac{x^2}{(x^3 - 1)^4} dx = \lim_{N \rightarrow \infty} \left. \frac{-1}{9(x^3 - 1)^3} \right|_2^N = -\frac{1}{9} \lim_{N \rightarrow \infty} \left. \frac{1}{(x^3 - 1)^3} \right|_2^N \\ &= -\frac{1}{9} \lim_{N \rightarrow \infty} \left( \frac{1}{(N^3 - 1)^3} - \frac{1}{(2^3 - 1)^3} \right) = -\frac{1}{9} \left( -\frac{1}{7^3} \right) = \frac{1}{3087} \end{aligned}$$

$$7. \int_2^{\infty} \frac{1}{\sqrt[3]{2x - 1}} dx$$

We first compute the indefinite integral by substitution. Let  $u = 2x - 1$ . Then  $du = 2dx$  and so  $dx = \frac{du}{2}$ .

$$\int \frac{1}{\sqrt[3]{2x - 1}} dx = \int \frac{1}{\sqrt[3]{u}} \frac{du}{2} = \frac{1}{2} \int u^{-1/3} du = \frac{1}{2} \frac{u^{2/3}}{2/3} + C = \frac{3}{4} u^{2/3} + C = \frac{3}{4} (2x - 1)^{2/3} + C$$

Now we are ready to evaluate the improper integral.

$$\begin{aligned} \int_2^{\infty} \frac{1}{\sqrt[3]{2x - 1}} dx &= \lim_{N \rightarrow \infty} \int_2^N \frac{1}{\sqrt[3]{2x - 1}} dx = \lim_{N \rightarrow \infty} \left. \frac{3}{4} (2x - 1)^{2/3} \right|_2^N = \frac{3}{4} \lim_{N \rightarrow \infty} (2x - 1)^{2/3} \Big|_2^N \\ &= \frac{3}{4} \lim_{N \rightarrow \infty} \left( (2N - 1)^{2/3} - (2 \cdot 2 - 1)^{2/3} \right) = \frac{3}{4} \lim_{N \rightarrow \infty} \left( (2N - 1)^{2/3} - 3^{2/3} \right) = \infty \end{aligned}$$

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