Sample Problems

Compute each of the following integrals.

1. $\int xe^x \, dx$
2. $\int xe^{-4x} \, dx$
3. $\int \ln x \, dx$

Practice Problems

1. $\int xe^{2x} \, dx$
2. $\int xe^{-3x} \, dx$
3. $\int_0^{\ln 2} xe^{-3x} \, dx$
4. $\int_0^{\infty} xe^{-3x} \, dx$
5. $\int x2^x \, dx$
6. $\int x^2 2^x \, dx$
7. $\int x \ln x \, dx$
8. $\int x^5 \ln x \, dx$
9. $\int_1^9 \frac{\ln x}{\sqrt{x}} \, dx$
10. $\int_1^{\infty} \frac{\ln x}{x^7} \, dx$
11. $\int \frac{x^3}{(x^2+2)^2} \, dx$

Sample Problems - Answers

1.) $xe^x - e^x + C$ 2.) $-\dfrac{1}{16}e^{-4x} - \dfrac{1}{4}xe^{-4x} + C$ 3.) $x\ln x - x + C$

Practice Problems - Answers

1.) $\dfrac{1}{2}xe^{2x} - \dfrac{1}{4}e^{2x} + C$ 2.) $-\dfrac{1}{9}e^{-3x} - \dfrac{1}{3}xe^{-3x} + C$ 3.) $\dfrac{7}{72} - \dfrac{1}{24}\ln 2$ 4.) $\dfrac{1}{9}$

5.) $\dfrac{2^x}{\ln 2}\left(x - \dfrac{1}{\ln 2}\right) + C$ 6.) $\dfrac{2^x}{\ln 2}\left(x^2 - \dfrac{2x}{\ln 2} + \dfrac{2}{\ln^2 2}\right) + C$ 7.) $\dfrac{1}{2}x^2\ln x - \dfrac{1}{4}x^2 + C$

8.) $\dfrac{1}{6}x^6\ln x - \dfrac{1}{36}x^6 + C$ 9.) $6\ln 9 - 8$ 10.) $\dfrac{1}{36}$ 11.) $\dfrac{1}{2}\ln\left(x^2+2\right) + \dfrac{1}{x^2+2} + C$

Sample Problems - Solutions

Compute each of the following integrals.

1. $\displaystyle\int xe^x \, dx$

Solution: We will integrate this by parts, using the formula

$$\int f'g = fg - \int fg'$$

Let $g(x) = x$ and $f'(x) = e^x$ Then we obtain g' and f by differentiation and integration.

$f(x) = e^x$	$g(x) = x$
$f'(x) = e^x$	$g'(x) = 1$

$$\begin{aligned} \int f'g &= fg - \int fg' \quad \text{becomes} \\ \int xe^x \, dx &= xe^x - \int e^x dx = xe^x - e^x + C \end{aligned}$$

We should check our result by differentiating the answer. Indeed,

$$(xe^x - e^x + C)' = e^x + xe^x - e^x = xe^x$$

2. $\displaystyle\int xe^{-4x} \, dx$

Solution: Let $g(x) = x$ and $f'(x) = e^{-4x}$ Then we obtain g' and f by differentiation and integration. To compute $f(x)$, we will use substitution. Let $u = -4x$ then $du = -4dx$ and so $dx = \dfrac{du}{-4}$.

$$f(x) = \int e^{-4x} \, dx = \int e^u \frac{du}{-4} = -\frac{1}{4}\int e^u \, du = -\frac{1}{4}e^u + C = -\frac{1}{4}e^{-4x} + C$$

We will choose $C = 0$ and so $f(x) = -\dfrac{1}{4}e^{-4x}$.

$f(x) = -\frac{1}{4}e^{-4x}$	$g(x) = x$
$f'(x) = e^{-4x}$	$g'(x) = 1$

$$\begin{aligned} \int f'g &= fg - \int fg' \quad \text{becomes} \\ \int xe^{-4x} \, dx &= -\frac{1}{4}xe^{-4x} - \int -\frac{1}{4}e^{-4x} \, dx = -\frac{1}{4}xe^{-4x} + \frac{1}{4}\int e^{-4x} \, dx = -\frac{1}{4}xe^{-4x} + \frac{1}{4}\left(-\frac{1}{4}e^{-4x}\right) + C \\ &= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x} + C \end{aligned}$$

We check our result by differentiating the answer.

$$\left(-\frac{1}{4}xe^{-4x}-\frac{1}{16}e^{-4x}+C\right)'=$$

$$\begin{aligned} &= -\frac{1}{4}\left(xe^{-4x}\right)'-\frac{1}{16}\left(e^{-4x}\right)'=-\frac{1}{4}\left(e^{-4x}+x\left(-4e^{-4x}\right)\right)-\frac{1}{16}\left(-4e^{-4x}\right) \\ &= -\frac{1}{4}e^{-4x}+xe^{-4x}+\frac{1}{4}e^{-4x}=xe^{-4x} \end{aligned}$$

3. $\displaystyle\int \ln x \, dx$

Solution: Let $g(x)=\ln x$ and $f'(x)=1$ Then we obtain g' and f by differentiation and integration.

$f(x)=x$	$g(x)=\ln x$
$f'(x)=1$	$g'(x)=\frac{1}{x}$

$$\begin{aligned} \int f'g &= fg-\int fg' \quad \text{becomes} \\ \int \ln x \, dx &= x\ln x-\int x\cdot\frac{1}{x}\,dx=x\ln x-\int 1\,dx=x\ln x-x+C \end{aligned}$$

We check our result by differentiating the answer.

$$(x\ln x-x+C)'=\ln x+x\cdot\frac{1}{x}-1=\ln x$$