Sample Problems

1. Consider the function $f(x) = -2x^3 + 9x^2 + 24$.
   a) Find all values of x for which f is increasing.
   b) Find all values of x for which f is decreasing.
   c) Find all values of x for which f has a relative maximum.
   d) Find all values of x for which f has a relative minimum.
   e) Sketch the graph of f.

2. Consider the function $f(x) = 6x^5 - 50x^3 - 120$.
   a) Find all values of x for which f is increasing.
   b) Find all values of x for which f is decreasing.
   c) Find all values of x for which f has a relative maximum.
   d) Find all values of x for which f has a relative minimum.
   e) Sketch the graph of f.

Practice Problems

For each of the following functions f given below,
a) Find all values of x for which f is increasing.
b) Find all values of x for which f is decreasing.
c) Find all values of x for which f has a relative maximum.
d) Find all values of x for which f has a relative minimum.
e) Sketch the graph of f.

1.) $f(x) = x^3 - 3x^2 + 6$

2.) $f(x) = -x^3 + 6x^2 + 36x - 60$

3.) $f(x) = -2x^3 + 12x^2 + 6x - 24$

4.) $f(x) = 3x^4 - 6x^2 + 8$

5.) $f(x) = -6x^5 + 20x^3$

6.) $f(x) = 5x^6 - 24x^5 + 30x^4 - 12$

7.) $f(x) = -3x^4 + 16x^3 + 1$

8.) $f(x) = -x^6 + 6x^4$

Sample Problems - Answers

1.) $f(x) = -2x^3 + 9x^2 + 24.$

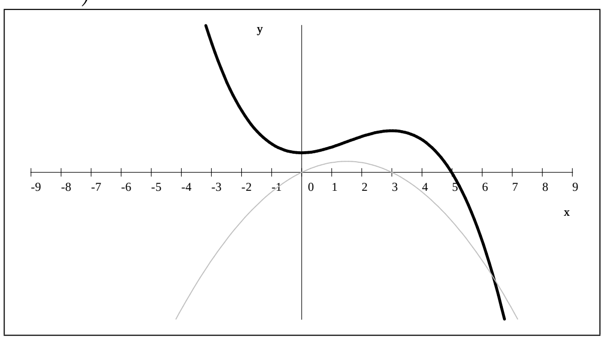
a) increasing on $(0, 3)$

b) decreasing on $(-\infty, 0)$ and on $(3, \infty)$

c) relative maximum at $x = 3$

d) relative minimum at $x = 0$

e)



2.) $f(x) = 6x^5 - 50x^3 - 120$

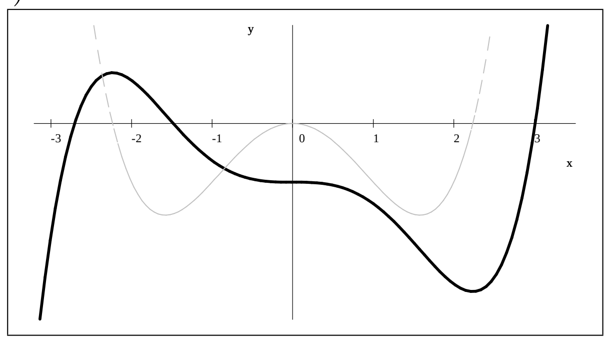
a) increasing on $\left(-\infty, -\sqrt{5}\right)$ and on $\left(\sqrt{5}, \infty\right)$

b) decreasing on $\left(-\sqrt{5}, \sqrt{5}\right)$

c) relative maximum at $x = -\sqrt{5}$

d) relative minimum at $x = \sqrt{5}$

e)



Practice Problems - Answers

1.) $f(x) = x^3 - 3x^2 + 6$

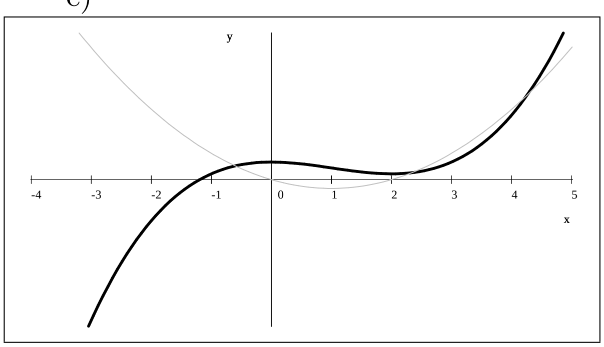
a) increasing on $(-\infty, 0)$ and on $(2, \infty)$

b) decreasing on $(0, 2)$

c) relative maximum at $x = 0$

d) relative minimum at $x = 2$

e)



2.) $f(x) = -x^3 + 6x^2 + 36x - 60$

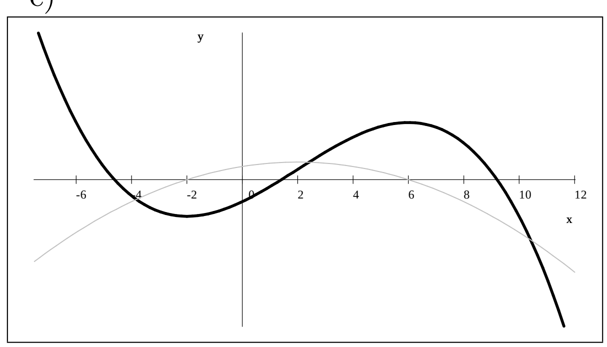
a) increasing on $(-2, 6)$

b) decreasing on $(-\infty, -2)$ and on $(6, \infty)$

c) relative maximum at $x = 6$

d) relative minimum at $x = -2$

e)



3.) $f(x) = -2x^3 + 12x^2 + 6x - 24$

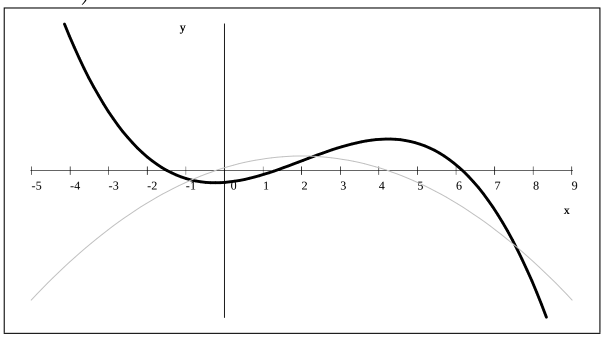
a) increasing on $\left(2-\sqrt{5}, 2+\sqrt{5}\right)$

b) decreasing on $\left(-\infty, 2-\sqrt{5}\right)$ and on $\left(2+\sqrt{5}, \infty\right)$

c) relative maximum at $x = 2+\sqrt{5}$

d) relative minimum at $x = 2-\sqrt{5}$

e)



4.) $f(x) = 3x^4 - 6x^2 + 8$

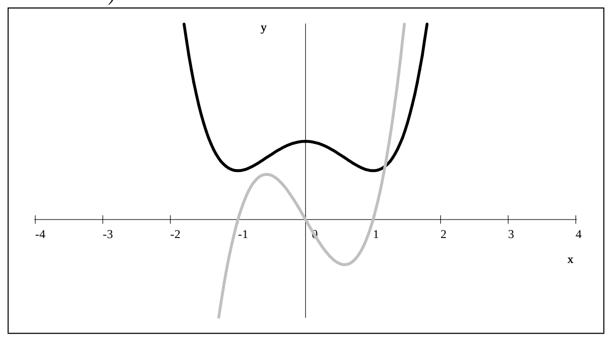
a) increasing on $(-1, 0)$ and on $(1, \infty)$

b) decreasing on $(-\infty, -1)$ and on $(0, 1)$

c) relative maximum at $x = 0$

d) relative minimum at $x = -1, 1$

e)



5.) $f(x) = -6x^5 + 20x^3$

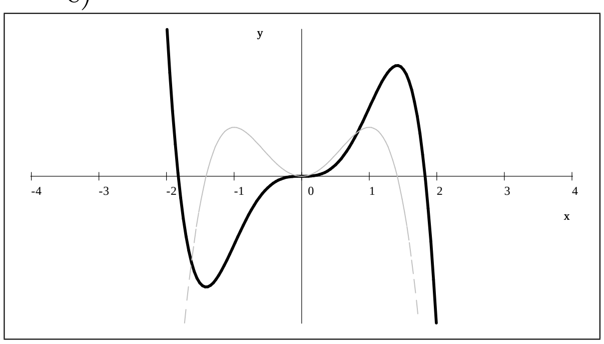
a) increasing on $\left(-\sqrt{2}, 0\right)$ and on $\left(0, \sqrt{2}\right)$

b) decreasing on $\left(-\infty, -\sqrt{2}\right)$ and on $\left(\sqrt{2}, \infty\right)$

c) relative maximum at $x = \sqrt{2}$

d) relative minimum at $x = -\sqrt{2}$

e)



6.) $f(x) = 5x^6 - 24x^5 + 30x^4 - 12$

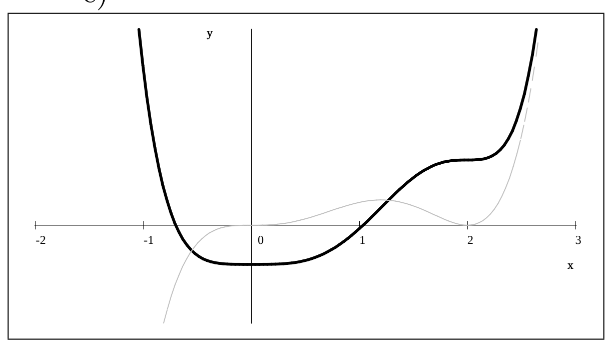
a) increasing on $(0, \infty)$

b) decreasing on $(-\infty, 0)$

c) no relative maximum

d) relative minimum at $x = 0$

e)



7.) $f(x) = -3x^4 + 16x^3 + 1$

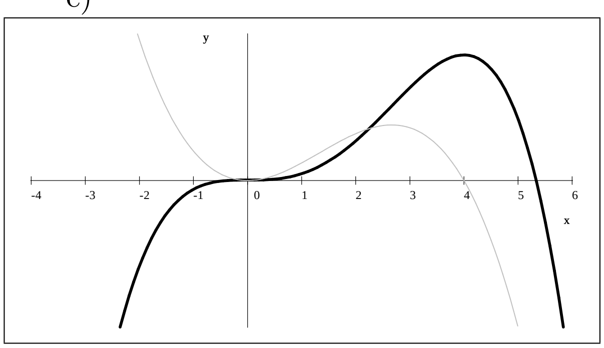
a) increasing on $(-\infty, 4)$

b) decreasing on $(4, \infty)$

c) relative maximum at $x = 4$

d) no relative minimum

e)



8.) $f(x) = -x^6 + 6x^4$

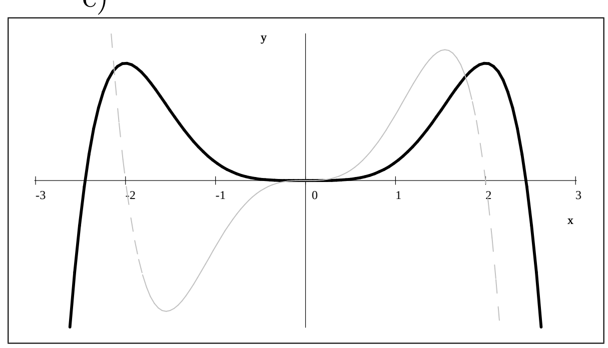
a) increasing on $(-\infty, -2)$ and on $(0, 2)$

b) decreasing on $(-2, 0)$ and on $(2, \infty)$

c) relative maximum at $x = -2, 2$

d) relative minimum at $x = 0$

e)



Sample Problems - Solutions

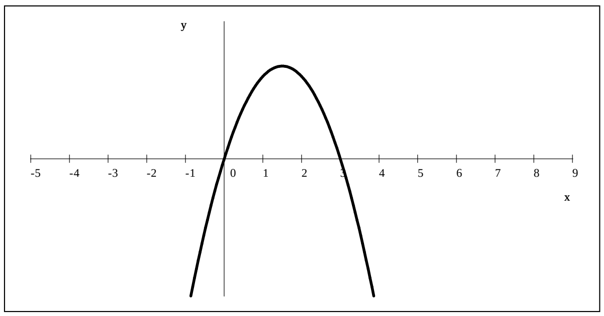
1.) Consider the function $f(x) = -2x^3 + 9x^2 + 24$.

a) Find all values of x for which f is increasing.

Solution: We compute f' first and then determine when f' is positive.

$$\begin{aligned} f(x) &= -2x^3 + 9x^2 + 24 \\ f'(x) &= -6x^2 + 18x = -6x(x-3) \end{aligned}$$

f' is a quadratic function with a negative leading coefficient. Thus its graph is a downward opening parabola. The $x-$intercepts are at $x = 0$ and $x = 3$. The graph of f' is thus



$f'(x) = -6x(x-3)$

When f' is positive, then f is increasing. We can see that f' is positive on $(0,3)$ and so f is increasing there. The answer is: f is increasing on $(0,3)$.

b) Find all values of x for which f is decreasing.

Solution: we differentiate f, and then factor and graph f' (see in part a). When f' is negative, then f is decreasing. From the graph we can see that f' is negative on $(-\infty, 0)$ and $(3, \infty)$ and so f is decreasing there. The answer is: f is decreasing on $(-\infty, 0)$ and on $(3, \infty)$.

c) Find all values of x for which f has a relative maximum.

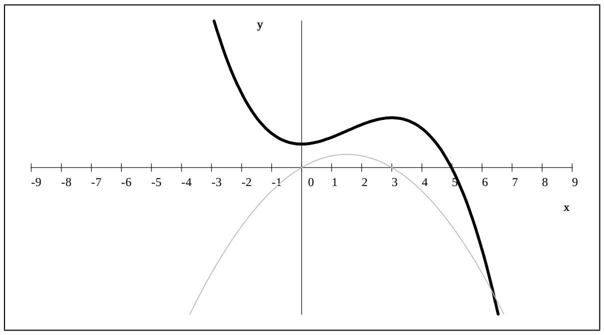
Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative maximum at x if f changes from increasing to decreasing at x. That happens when f' changes sign from positive to negative. Based on the answers for parts a) and b), this happens at $x = 3$. Thus f has a relative maximum at $x = 3$.

d) Find all values of x for which f has a relative minimum.

Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative minimum at x if f changes from decreasing to increasing at x. That is the same as f' changing from negative to positive. Based on the answers for parts a) and b), this happens at $x = 0$. Thus f has a relative minimum at $x = 0$.

e) Sketch the graph of f.

Solution: We evaluate f at the relative minimum and maximum.
$f(0) = 24$ and $f(3) = 51$. Note that the only $x-$intercept of f is irrational and it would take solving a cubic equation to find its exact value. If needed, we may evaluate f at additional points. In case of polynomials, the $y-$intercept is a very easy one.



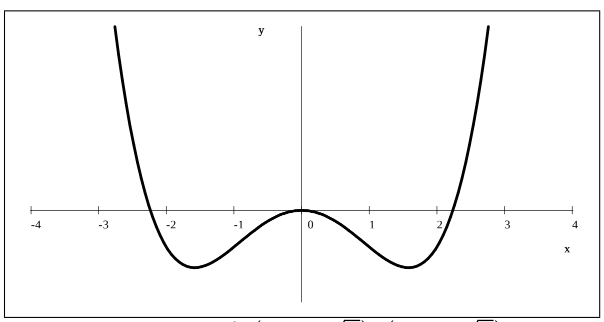
2.) Consider the function $f(x) = 6x^5 - 50x^3 - 120$.

a) Find all values of x for which f is increasing.

Solution: Let us compute f' first.

$$\begin{aligned} f(x) &= 6x^5 - 50x^3 - 120 \\ f'(x) &= 30x^4 - 150x^2 = 30x^2\left(x^2 - 5\right) = 30x^2\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right) \end{aligned}$$

f' is a degree four polynomial function with a positive leading coefficient and $x-$intercepts at $x = -\sqrt{5}, 0, \sqrt{5}$. The graph of f' is thus



$f'(x) = x^2\left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)$

When f' is positive, then f is increasing. We can see that f' is positive on $\left(-\infty, -\sqrt{5}\right)$ and $\left(\sqrt{5}, \infty\right)$ so f is increasing there. The answer is: f is increasing on $\left(-\infty, -\sqrt{5}\right)$ and on $\left(\sqrt{5}, \infty\right)$.

b) Find all values of x for which f is decreasing.

Solution: we differentiate f, and then factor and graph f' (see in part a). When f' is negative, then f is decreasing. From the graph we can see that f' is negative on $\left(-\sqrt{5}, 0\right)$ and on $\left(0, \sqrt{5}\right)$ and so f is decreasing there.
In this particular case, f is decreasing on the entire interval $\left(-\sqrt{5}, \sqrt{5}\right)$. One must be careful when making this step. For example, the function $g(x) = \dfrac{1}{x}$ is decreasing on $(-\infty, 0)$ and $(0, \infty)$ but is not decreasing on $(-\infty, \infty)$.

c) Find all values of x for which f has a relative maximum.

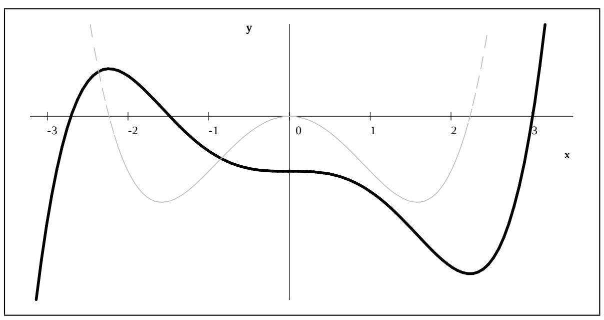
Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative maximum at x if f changes from increasing to decreasing at x. That happens when f' changes sign from positive to negative. Based on the answers for parts a) and b), this happens at $x = -\sqrt{5}$. Thus f has a relative maximum at $x = -\sqrt{5}$.

d) Find all values of x for which f has a relative minimum.

Solution: Since f is a polynomial, it is continuous everywhere. Thus f has a relative minimum at x if f changes from decreasing to increasing at x. That happens when f' changes sign from negative to positive. Based on the answers for parts a) and b), this happens at $x = \sqrt{5}$. Thus f has a relative minimum at $x = \sqrt{5}$.
What about the zero of f' at $x = 0$? Does f have a relative maximum or a minimum at $x = 0$? The answer is: neither. The derivative f' is negative on an interval before $x = 0$ and also on an interval after $x = 0$. Consequently, f is decreasing before and after $x = 0$ and so there can be neither a relative maximum nor a relative minimum there. This situation is similar to $g(x) = x^3$ at $x = 0$. Although its derivative, $g'(x) = 3x^2$ has a zero at $x = 0$, there is no change in sign around the zero there, and so $g(x) = x^3$ does not have a relative minimum or maximum at $x = 0$.

e) Sketch the graph of f.

Solution: We evaluate f at the relative minimum and maximum.
$f(0) = -120$, $f\left(-\sqrt{5}\right) = -120 + 100\sqrt{5} \approx 103.606\,8$ and $f\left(\sqrt{5}\right) = -120 - 100\sqrt{5} \approx -343.606\,8$.



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