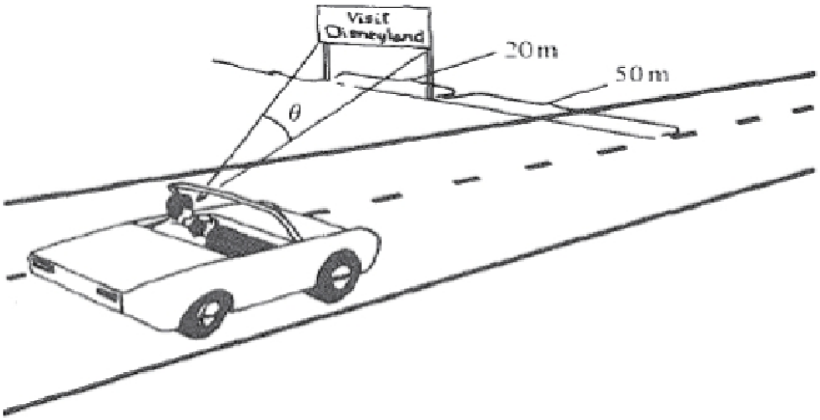
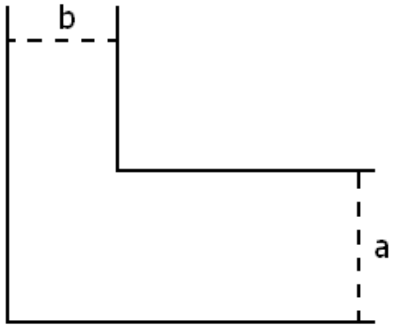
1. A searchlight 100 meters from a road is tracking a car moving at 100 kilometers per hour. At what rate (in degrees per second) is the searchlight turning when the car is 141 meters away?

2. At what position on the road is the angle θ maximized?



3. How long is the longest straight rod that can be carried through the corner shown on the picture below?
   a) Assume that $a = 10$ and $b = 6$
   b) Solve the problem in general, using a and b.



4. Particle A is moving in the plane according to $x = 3\sin 3t$ and $y = 3\cos 3t$ and particle B is moving according to $x = 3\cos 2t$ and $y = 3\sin 2t$. Find the maximum distance between A and B.

5. Two trains, each 50 meters long, are moving away from the intersection point of perpendicular tracks at the same speed. Where are the trains when train A subtends the largest angle as seen from the front of train B?

6. Where is the function $f(x) = \sin^3 x$ concave up? Concave down?

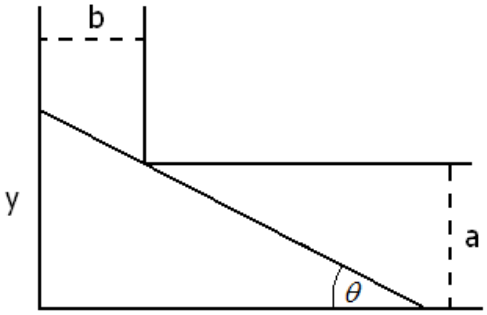
Answers

1. $\frac{1}{100}\left(\frac{100000}{3600}\right)\left(\frac{100}{141}\right)^2 \approx 0.139\,72\left(\frac{180}{\pi}\right)\text{rad} = \frac{8.005\,4^\circ}{\text{s}}$

2. $\sqrt{3500} \approx 59.161$

3. a) $\theta = \tan^{-1}\left(\sqrt[3]{\frac{10}{6}}\right) \approx 49.855^\circ \quad L = \frac{10}{\sin(49.855^\circ)} + \frac{6}{\cos(49.855^\circ)} \approx 22.388$

   b) $\theta = \tan^{-1}\left(\sqrt[3]{\frac{a}{b}}\right) \quad L = \frac{a}{\sin\theta} + \frac{b}{\cos\theta}$ So, $L = \frac{a}{\sin\left(\tan^{-1}\left(\sqrt[3]{\frac{a}{b}}\right)\right)} + \frac{b}{\cos\left(\tan^{-1}\left(\sqrt[3]{\frac{a}{b}}\right)\right)}$



4. 6

5. right at the start, both at the station

6. Concave up where $\cos x > 0$ - that is, $-\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$

   Concave up where $\cos x < 0$ - that is, $\frac{\pi}{2} + 2k\pi < x < \frac{3\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$