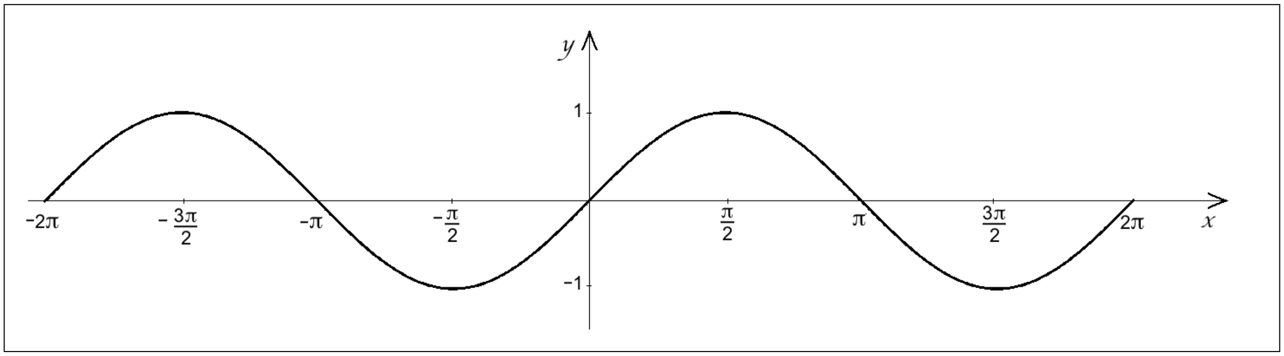
$f(x) = \sin x$

domain: $\mathbb{R}$

range: $[-1, 1]$

periodic with period 2π: for all x, $\sin(x + 2\pi) = \sin x$

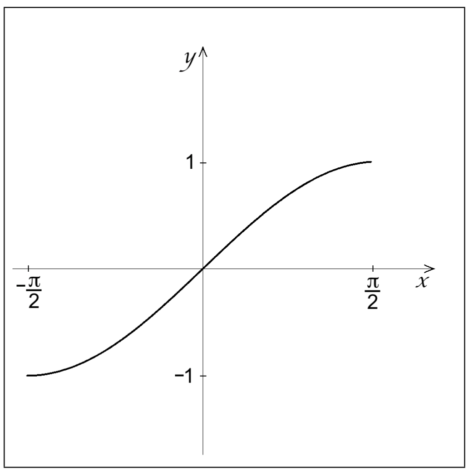
odd function: for all x, $\sin(-x) = -\sin x$



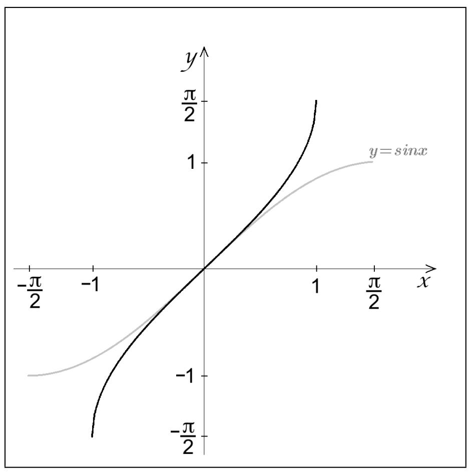
inverse function: $y = \sin^{-1} x = \arcsin x$

domain: $[-1, 1]$

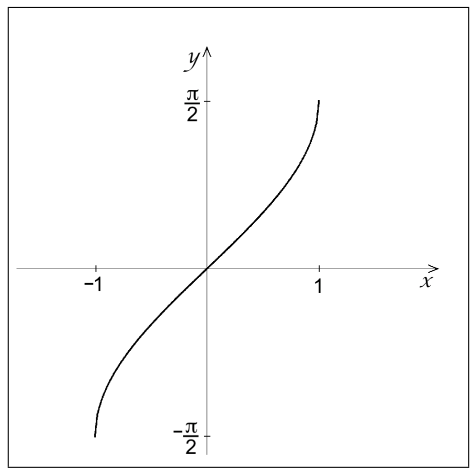
range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Restrict the domain of $\sin x$ to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Reverse function assignment

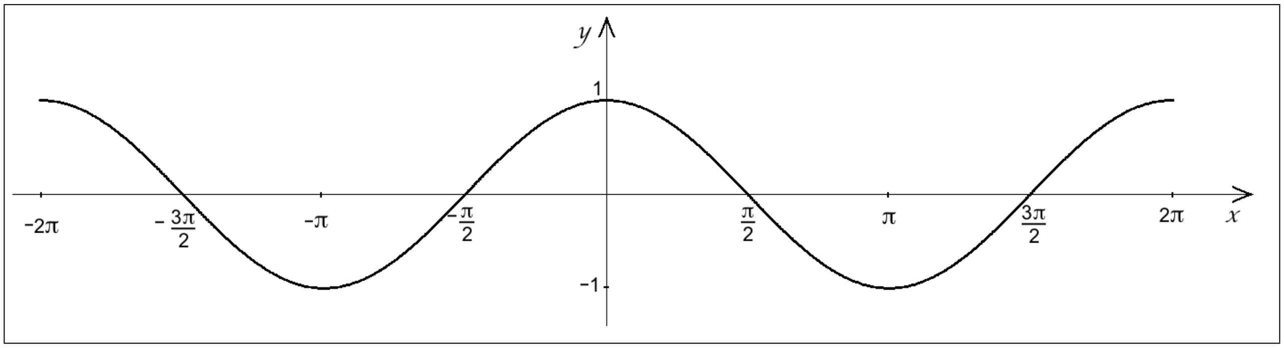


The graph of $y = \sin^{-1} x$

$$\boxed{f(x) = \cos x}$$

domain: $\mathbb{R}$
range: $[-1, 1]$

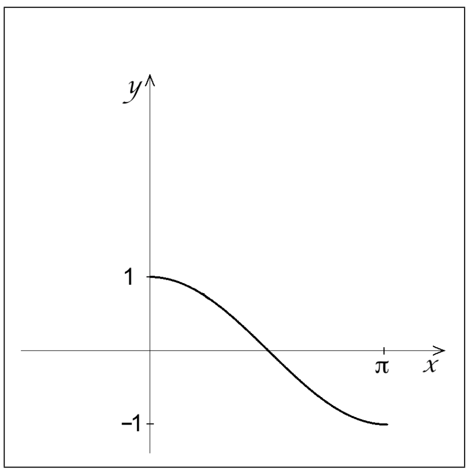
periodic with period 2π: for all x, $\cos(x + 2\pi) = \cos x$
even function: for all x, $\cos(-x) = \cos x$



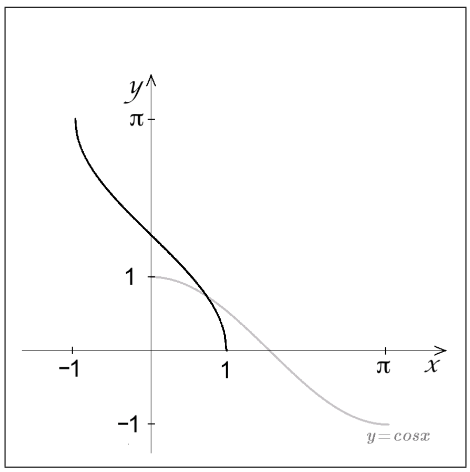
inverse function: $y = \cos^{-1} x = \arccos x$

domain: $[-1, 1]$

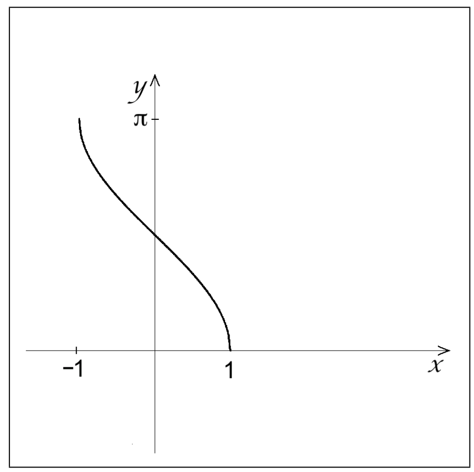
range: $[0, \pi]$



Restrict the domain of $\cos x$ to $[0, \pi]$



Reverse function assignment



The graph of $y = \cos^{-1} x$

Theorem:

$$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2} \quad \text{and} \quad \cos^{-1} x + \cos^{-1}(-x) = \pi$$

$$\boxed{f(x) = \tan x = \frac{\sin x}{\cos x}}$$

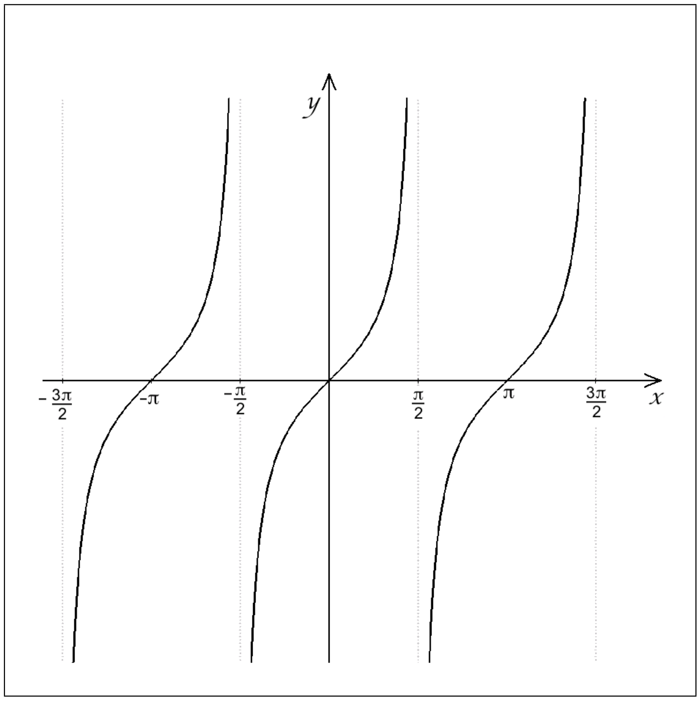
domain: $x \neq \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

range: $\mathbb{R}$

periodic with period π: for all x, $\tan(x + \pi) = \tan x$

vertical asymptotes at $x = \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

odd function: for all x, $\tan(-x) = -\tan x$



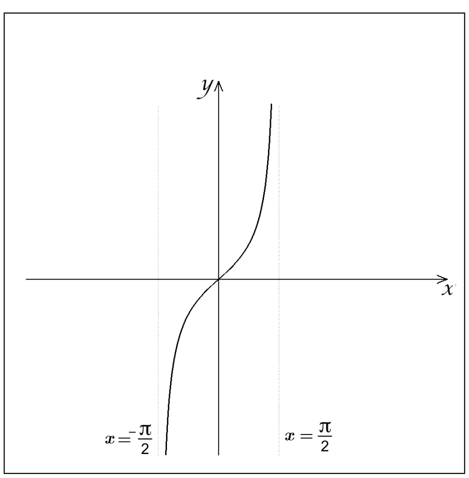
inverse function: $y = \tan^{-1} x = \arctan x$

domain: $\mathbb{R}$

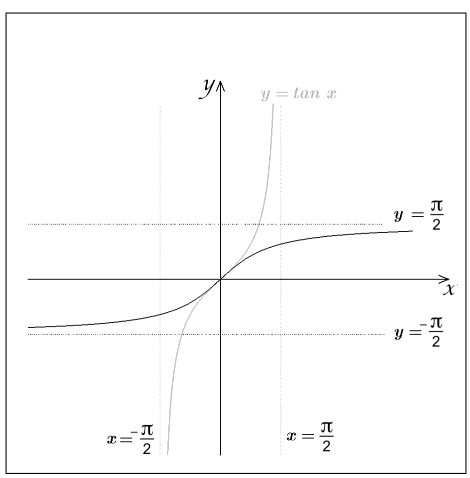
range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\lim\limits_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$ and $\lim\limits_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$

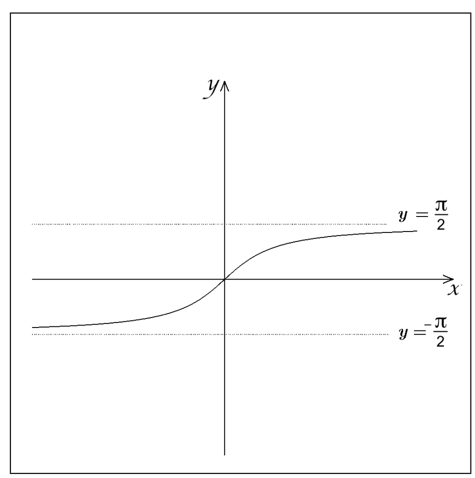
horizontal asymptotes: $y = \pm\frac{\pi}{2}$



Restrict the domain of $\tan x$ to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



Reverse function assignment



The graph of $y = \tan^{-1} x$

$$f(x) = \csc x = \frac{1}{\sin x}$$

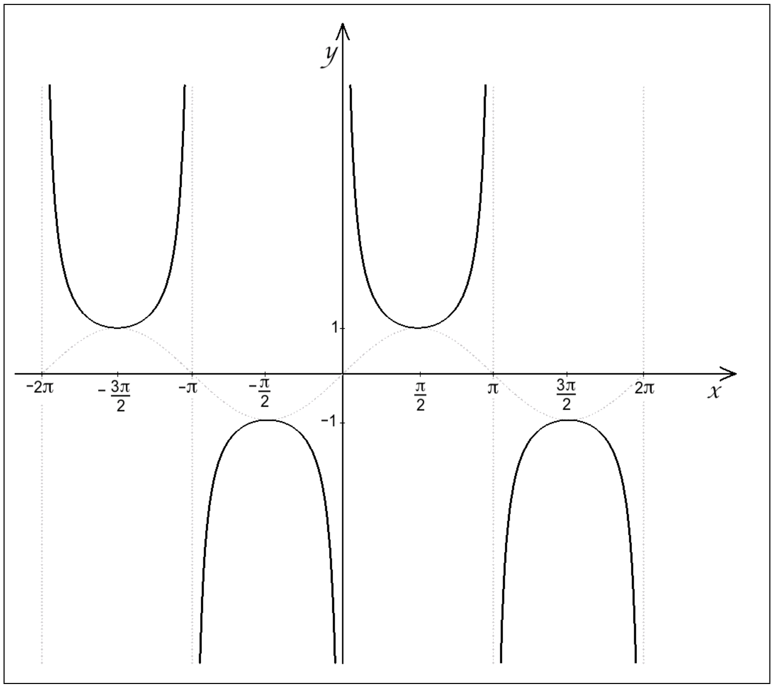
domain: $x \neq k\pi$

vertical asymptotes at $x = k\pi$ where $k \in \mathbb{Z}$

range: $(-\infty, -1] \cup [1, \infty)$

periodic with period 2π: for all x, $\csc(x + 2\pi) = \csc x$

odd function: for all x, $\csc(-x) = -\csc x$



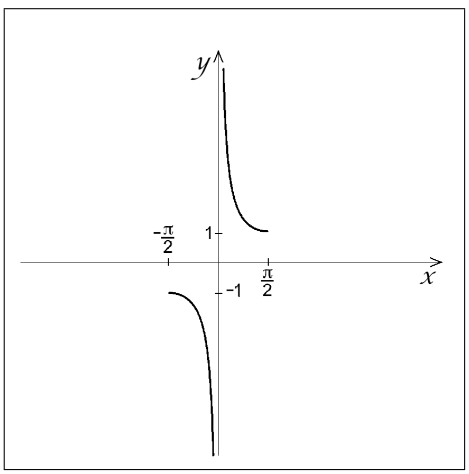
inverse function: $y = \csc^{-1} x = \operatorname{arccsc} x$

domain: $(-\infty, -1] \cup [1, \infty)$

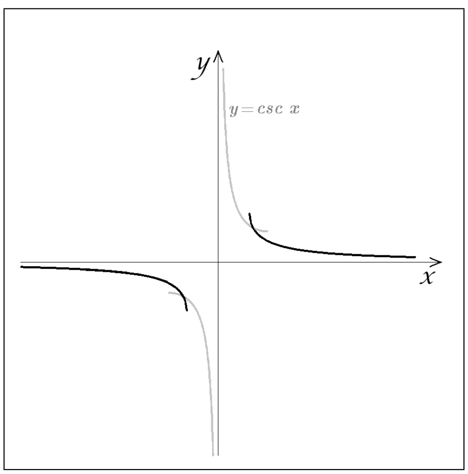
range: $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

$\lim_{x \to -\infty} \csc^{-1} x = 0$ and $\lim_{x \to \infty} \csc^{-1} x = 0$

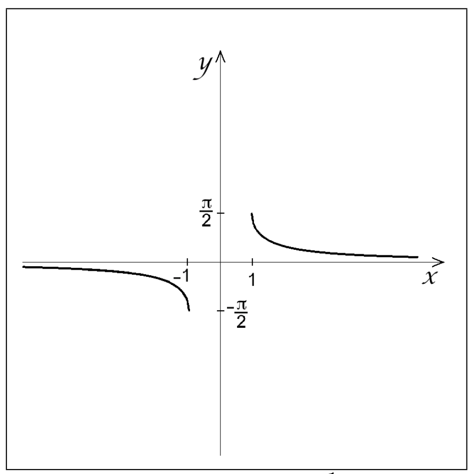
horizontal asymptote: $y = 0$



Restrict the domain of $\csc x$ to $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Reverse function assignment



The graph of $y = \csc^{-1} x$

$$\boxed{f(x) = \sec x = \frac{1}{\cos x}}$$

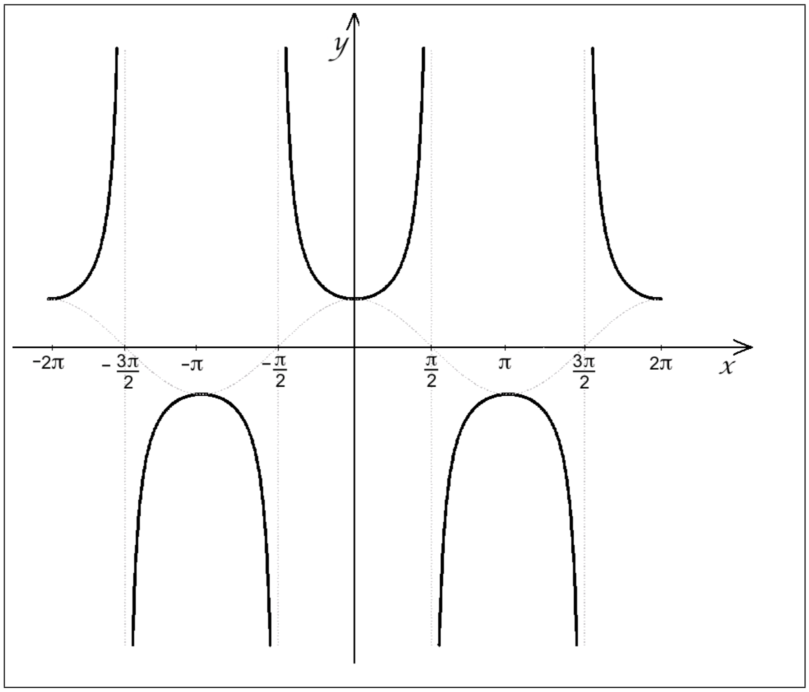
domain: $x \neq \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

range: $(-\infty, -1] \cup [1, \infty)$

periodic with period 2π: for all x, $\sec(x + 2\pi) = \sec x$

vertical asymptotes at $x = \frac{\pi}{2} + k\pi$ where $k \in \mathbb{Z}$

even function: for all x, $\sec(-x) = \sec x$



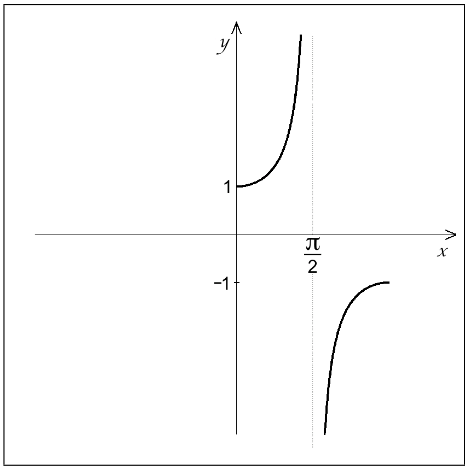
inverse function: $y = \sec^{-1} x = \operatorname{arcsec} x$

domain: $(-\infty, -1] \cup [1, \infty)$

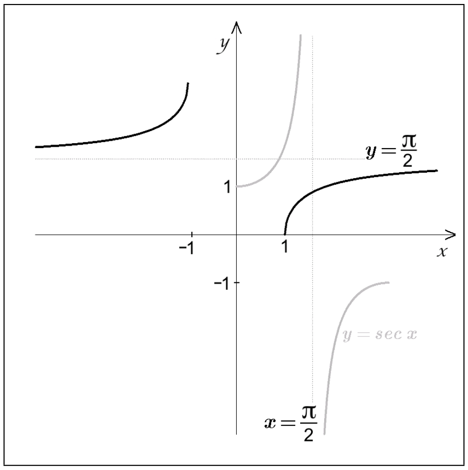
range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

$\lim_{x \to -\infty} \sec^{-1} x = \frac{\pi}{2}$ and $\lim_{x \to \infty} \sec^{-1} x = \frac{\pi}{2}$

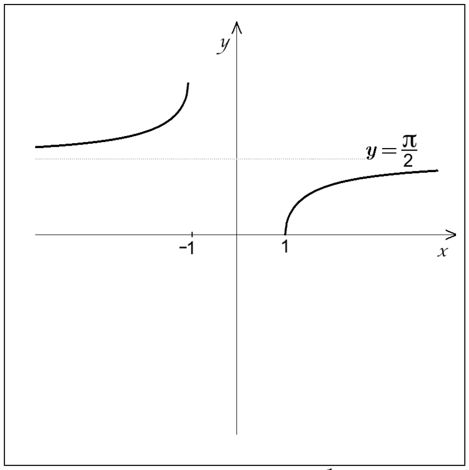
horizontal asymptote: $y = \frac{\pi}{2}$



Restrict the domain of $\sec x$ to $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



Reverse function assignment



The graph of $y = \sec^{-1} x$

 Last revised: January 24, 2013

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$

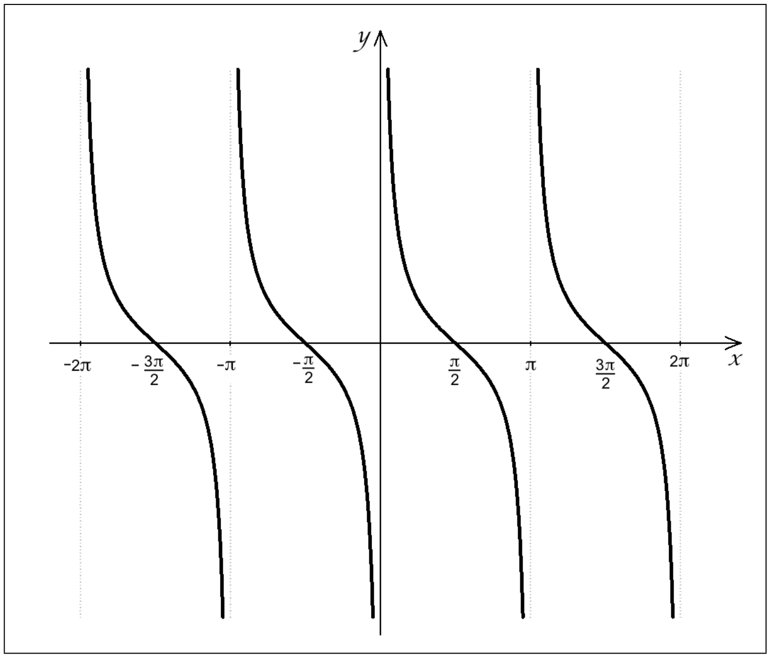
domain: $x \neq k\pi$

range: $\mathbb{R}$

periodic with period π: for all x, $\cot(x+\pi) = \csc x$

vertical asymptotes at $x = k\pi$ where $k \in \mathbb{Z}$

odd function: for all x, $\cot(-x) = -\cot x$



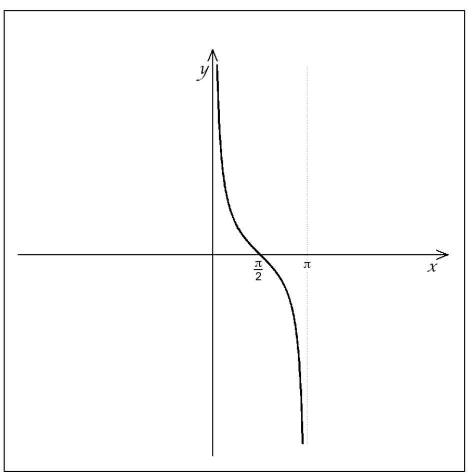
inverse function: $y = \cot^{-1} x = \operatorname{arccot} x$

domain: $\mathbb{R}$

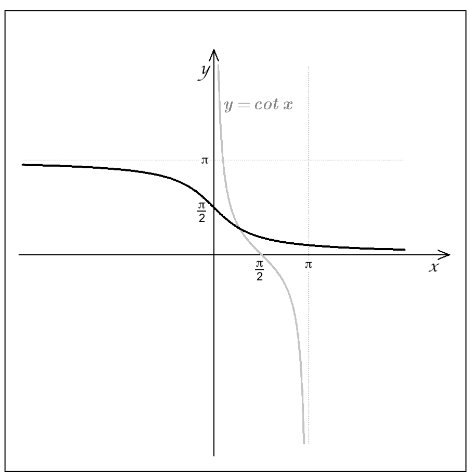
range: $(0, \pi)$

$\lim_{x\to-\infty} \cot^{-1} x = \pi$ and $\lim_{x\to\infty} \cot^{-1} x = 0$

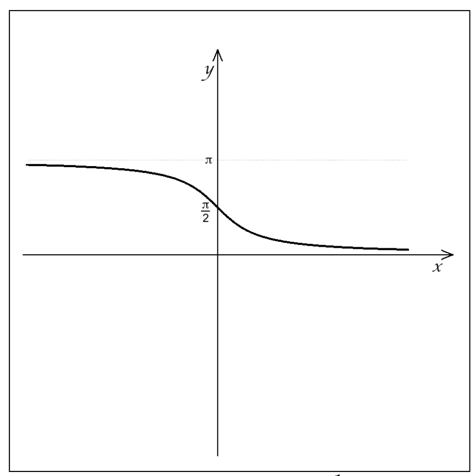
horizontal asymptote: $y = 0$ and $y = \pi$



Restrict the domain of $\cot x$ to $(0, \pi)$.



Reverse function assignment



The graph of $y = \cot^{-1} x$

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