Sample Problems

Differentiate each of the following functions.

1. $f(x) = -(5x+1)^{12}$
2. $f(x) = \sqrt{x^4 - 1}$
3. $f(x) = \dfrac{(x+3)^4 + (x-3)^4}{2}$
4. $f(x) = \dfrac{(x+3)^4 - (x-3)^4}{2}$
5. $f(x) = \log_x(x^2+1)$
6. $f(x) = \ln(\ln(x^5))$
7. $f(x) = (x-1)^{10}(4-x)^5$
8. $f(x) = \dfrac{5x-1}{5x+1}$

Practice Problems

Differentiate each of the following functions. Assume that a, b, and c are constants.

1. $f(x) = \log_7(x^2-1)$
2. $f(x) = \dfrac{x^5 - 3x^2 + 19}{2x}$
3. $f(x) = \dfrac{x^2}{2x+1}$
4. $f(x) = xe^{-5x}$
5. $f(x) = \dfrac{x+7}{x-7}$
6. $f(x) = \sqrt[3]{5x^2 - 7x + 11^x - e^8}$
7. $f(x) = -\dfrac{2}{3}(x^7 + 2x - 15)$
8. $f(x) = \dfrac{a}{b + \sqrt{x}}$
9. $f(x) = \dfrac{1}{4}xe^{4x} - \dfrac{1}{16}e^{4x}$
10. $f(x) = \dfrac{e^x - e^{-x}}{2}$
11. $f(x) = \dfrac{3-2x}{3+2x}$
12. $f(x) = (\ln(3x) - \ln(5x))^4$
13. $f(x) = \dfrac{x^2+6x+5}{x+1}$
14. $f(x) = 2^{3x} \cdot \log_5 2x$
15. $f(x) = \left(\dfrac{1}{x} - \dfrac{1}{x^2}\right)(2x^3+4)$
16. $f(x) = \dfrac{1}{\ln(x^7 - 9x^2 + 1)}$
17. $f(x) = \dfrac{ax+b}{bx+c}$
18. $f(x) = x\ln x - x$
19. $f(x) = \dfrac{8x^2 - 20x + 1}{4x - 10}$
20. $f(x) = \dfrac{2}{3}(x+1)\sqrt{x+1}$

Sample Problems - Answers

1. $f'(x) = -60\left(5x+1\right)^{11}$

2. $f'(x) = \dfrac{2x^3}{\sqrt{x^4-1}}$

3. $f'(x) = 4x^3 + 108x$

4. $f'(x) = 36x^2 + 108$

5. $f'(x) = \dfrac{\dfrac{2x\ln x}{x^2+1} - \dfrac{\ln\left(x^2+1\right)}{x}}{\left(\ln x\right)^2}$

6. $f'(x) = \dfrac{1}{x\ln x}$

7. $f'(x) = -15\left(x-1\right)^9\left(x-3\right)\left(x-4\right)^4$

8. $f'(x) = \dfrac{10}{\left(5x+1\right)^2}$

Practice Problems - Answers

1. $f'(x) = \dfrac{2x}{\left(x^2-1\right)\ln 7}$

2. $f'(x) = 2x^3 - \dfrac{19}{2x^2} - \dfrac{3}{2}$

3. $f'(x) = \dfrac{2x^2+2x}{\left(2x+1\right)^2}$

4. $f'(x) = e^{-5x} - 5xe^{-5x}$

5. $f'(x) = -\dfrac{14}{\left(x-7\right)^2}$

6. $f'(x) = \dfrac{10x + \left(\ln 11\right)11^x - 7}{3\left(5x^2-7x+11^x-e^8\right)^{2/3}}$

7. $f'(x) = -\dfrac{14}{3}x^6 - \dfrac{4}{3}$

8. $f'(x) = -\dfrac{a}{2\sqrt{x}\left(b+\sqrt{x}\right)^2}$

9. $f'(x) = xe^{4x}$

10. $f'(x) = \dfrac{e^x + e^{-x}}{2}$

11. $f'(x) = -\dfrac{12}{\left(2x+3\right)^2}$

12. $f'(x) = 0$

13. $f'(x) = 1$

14. $f'(x) = 3\dfrac{\ln 2}{\ln 5}\left(\ln x + \ln 2\right)2^{3x} + \dfrac{2^{3x}}{x\ln 5}$

15. $f'(x) = 4x - \dfrac{4}{x^2} + \dfrac{8}{x^3} - 2$

16. $f'(x) = -\dfrac{7x^6 - 18x}{\left(x^7-9x^2+1\right)\ln^2\left(x^7-9x^2+1\right)}$

17. $f'(x) = \dfrac{ac-b^2}{\left(bx+c\right)^2}$

18. $f'(x) = \ln x$

19. $f'(x) = \dfrac{8x^2-40x+49}{4x^2-20x+25} = 2 - \dfrac{1}{\left(2x-5\right)^2}$

20. $f'(x) = \sqrt{x+1}$

Sample Problems - Solutions

Differentiate each of the following functions.

1. $f(x) = -(5x+1)^{12}$ $f'(x) = -60(5x+1)^{11}$
   Solution: We apply the chain rule.
   $$f'(x) = -12 \cdot (5x+1)^{11} \cdot (5) = -60(5x+1)^{11}$$

2. $f(x) = \sqrt{x^4 - 1}$ $f'(x) = \dfrac{2x^3}{\sqrt{x^4-1}}$
   Solution: We re-write f with exponential notation and differentiate it via the chain rule.
   $$\begin{aligned} f(x) &= \left(x^4 - 1\right)^{1/2} \\ f'(x) &= \frac{1}{2}\left(x^4-1\right)^{-1/2} \cdot \left(4x^3\right) = \frac{2x^3}{\sqrt{x^4-1}} \end{aligned}$$

3. $f(x) = \dfrac{(x+3)^4 + (x-3)^4}{2}$ $f'(x) = 4x^3 + 108x$
   Solution: We re-write f and then apply the chain rule.
   $$\begin{aligned} f(x) &= \frac{(x+3)^4 + (x-3)^4}{2} = \frac{1}{2}(x+3)^4 + \frac{1}{2}(x-3)^4 \\ f'(x) &= \frac{1}{2}(4)(x+3)^3(1) + \frac{1}{2}(4)(x-3)^3(1) = 2(x+3)^3 + 2(x-3)^3 \end{aligned}$$
   Thus $f'(x) = 2(x+3)^3 + 2(x-3)^3$ or, as a polynomial,
   $$\begin{aligned} f'(x) &= 2(x+3)^3 + 2(x-3)^3 = 2\left(x^3 + 9x^2 + 27x + 27 + x^3 - 9x^2 + 27x - 27\right) \\ &= 2\left(2x^3 + 54x\right) = 4x^3 + 108x \end{aligned}$$

4. $f(x) = \dfrac{(x+3)^4 - (x-3)^4}{2}$ $f'(x) = 36x^2 + 108$
   Solution: We re-write f and then apply the chain rule.
   $$\begin{aligned} f(x) &= \frac{(x+3)^4 - (x-3)^4}{2} = \frac{1}{2}(x+3)^4 - \frac{1}{2}(x-3)^4 \\ f'(x) &= \frac{1}{2}(4)(x+3)^3(1) - \frac{1}{2}(4)(x-3)^3(1) = 2(x+3)^3 - 2(x-3)^3 \end{aligned}$$
   Thus $f'(x) = 2(x+3)^3 - 2(x-3)^3$ or, as a polynomial,
   $$\begin{aligned} f'(x) &= 2(x+3)^3 - 2(x-3)^3 = 2\left(x^3 + 9x^2 + 27x + 27 - \left(x^3 - 9x^2 + 27x - 27\right)\right) \\ &= 2\left(x^3 + 9x^2 + 27x + 27 - x^3 + 9x^2 - 27x + 27\right) \\ &= 2\left(18x^2 + 54\right) = 36x^2 + 108 \end{aligned}$$

5. $f(x) = \log_x(x^2+1)$ $f'(x) = \dfrac{\dfrac{2x\ln x}{x^2+1} - \dfrac{\ln(x^2+1)}{x}}{(\ln x)^2}$

Solution: We re-write f and apply the quotient rule. To differentiate $\ln(x^2+1)$, we apply the chain rule.

$$f'(x) = \left(\log_x\left(x^2+1\right)\right)' = \left(\frac{\ln(x^2+1)}{\ln x}\right)' = \frac{\left(\ln(x^2+1)\right)'\ln x - \ln(x^2+1)(\ln x)'}{(\ln x)^2}$$

$$= \frac{\left(\dfrac{1}{x^2+1}(2x)\right)'\ln x - \ln(x^2+1)\left(\dfrac{1}{x}\right)}{(\ln x)^2} = \frac{\dfrac{2x\ln x}{x^2+1} - \dfrac{\ln(x^2+1)}{x}}{(\ln x)^2}$$

6. $f(x) = \ln(\ln(x^5))$ $f'(x) = \dfrac{1}{x\ln x}$

Solution: We re-write f using properties of logarithms and apply the chain rule.

$$f(x) = \ln\left(\ln\left(x^5\right)\right) = \ln(5\ln x) = \ln 5 + \ln\ln x$$

$$f'(x) = (\ln 5 + \ln\ln x)' = 0 + \frac{1}{\ln x}\cdot\frac{1}{x} = \frac{1}{x\ln x}$$

7. $f(x) = (x-1)^{10}(4-x)^5$ $f'(x) = -15(x-1)^9(x-3)(x-4)^4$

Solution: We first factor out the leading coefficients. Then we differentiate f using the product rule.

$$f(x) = (x-1)^{10}(4-x)^5 = -(x-1)^{10}(x-4)^5$$

$$f'(x) = -\left(10(x-1)^9(x-4)^5 + (x-1)^{10}(5)(x-4)^4\right) = -\left(10(x-1)^9(x-4)^5 + 5(x-1)^{10}(x-4)^4\right)$$

From this expression, we can factor out $5(x-1)^9(x-4)^4$

$$f'(x) = -5(x-1)^9(x-4)^4(2(x-4)+(x-1)) = -5(x-1)^9(x-4)^4(3x-9)$$

$$= -5(x-1)^9(x-4)^4(3)(x-3) = -15(x-1)^9(x-3)(x-4)^4$$

8. $f(x) = \dfrac{5x-1}{5x+1}$ $f'(x) = \dfrac{10}{(5x+1)^2}$

Solution: Method 1

Although you may use the quotient rule, it can be avoided with a little algebraic transformation before the calculus:

$$f(x) = \frac{5x-1}{5x+1} = \frac{5x+1-1-1}{5x+1} = \frac{5x+1}{5x+1} - \frac{2}{5x+1} = 1 - \frac{2}{5x+1} = 1 - 2(5x+1)^{-1}$$

$$f'(x) = \left(1-2(5x+1)^{-1}\right)' = 0 - 2(-1)(5x+1)^{-2}(5) = 10(5x+1)^{-2} = \frac{10}{(5x+1)^2}$$

Method 2

Apply the quotient rule.

$$f'(x) = \left(\frac{5x-1}{5x+1}\right)' = \frac{(5x-1)'(5x+1) - (5x-1)(5x+1)'}{(5x-1)^2}$$

$$= \frac{5(5x+1) - 5(5x-1)}{(5x-1)^2} = \frac{25x+5-25x+5}{(5x-1)^2} = \frac{10}{(5x-1)^2}$$