Sample Problems

1. Find an equation for the inverse for each of the function given below.

   a) $f(x) = 3x + 2$

   b) $f(x) = (5x-1)^3$

   c) $f(x) = \dfrac{x+4}{3x-5}$

   d) $f(x) = \log_5(2x-1)$

   e) $f(x) = e^{5x-1}$

2. Prove that the inverse of a linear function is also linear and the two slopes are reciporcals of each other.

Practice Problems

1. Find an equation for the inverse of each of the following functions.

   a) $f(x) = 3^{5x-1}$

   b) $f(x) = \dfrac{2x-7}{3x+5}$

   c) $f(x) = \ln(2x-1)$

   d) $f(x) = \dfrac{1}{3x+1}$

   e) $f(x) = \dfrac{3\sqrt{x}-5}{2}$

   f) $f(x) = \log_3\left(\dfrac{1}{2}x - 7\right)$

   g) $f(x) = \dfrac{2x+5}{3x-2}$

Sample Problems - Answers

1. a) $f^{-1}(x) = \frac{1}{3}(x-2)$ b) $f^{-1}(x) = \frac{1}{5}(\sqrt[3]{x}+1)$ c) $f^{-1}(x) = \frac{5x+4}{3x-1}$

   d) $f^{-1}(x) = \frac{1}{2}(5^x+1)$ e) $f^{-1}(x) = \frac{1}{5}(\ln x+1)$

2. see solutions

Practice Problems - Answers

1. a) $f^{-1}(x) = \frac{1}{5}(\log_3 x+1)$ b) $f^{-1}(x) = \frac{5x+7}{-3x+2}$ c) $f^{-1}(x) = \frac{1}{2}(e^x+1)$

   d) $f^{-1}(x) = \frac{-x+1}{3x}$ e) $f^{-1}(x) = \left(\frac{2x+5}{3}\right)^2$ f) $f^{-1}(x) = 2\cdot 3^x+14$

   g) $f^{-1}(x) = \frac{2x+5}{3x-2}$

Sample Problems - Solutions

1. Find an equation for the inverse for each of the function given below.

   a) $f(x) = 3x+2$
   Solution: First we drop the function notation and write y instead of $f(x)$. Then we solve for x and finally, swap x and y.

$$\begin{aligned} y &= 3x+2 && \text{subtract 2} \\ y-2 &= 3x && \text{divide by 3} \\ \frac{y-2}{3} &= x \quad \text{swap } x \text{ and } y && y=\frac{x-2}{3} \qquad f^{-1}(x)=\frac{1}{3}(x-2) \end{aligned}$$

   b) $f(x) = (5x-1)^3$
   Solution: First we drop the function notation and write y instead of $f(x)$. Then we solve for x and finally, swap x and y.

$$\begin{aligned} y &= (5x-1)^3 && \text{take 3rd root of both sides} \\ \sqrt[3]{y} &= 5x-1 && \text{add 1} \\ \sqrt[3]{y}+1 &= 5x && \text{divide by 5} \\ \frac{\sqrt[3]{y}+1}{5} &= x \quad \text{swap } x \text{ and } y && y=\frac{1}{5}\left(\sqrt[3]{x}+1\right) \qquad f^{-1}(x)=\frac{1}{5}\left(\sqrt[3]{x}+1\right) \end{aligned}$$

c) $f(x) = \dfrac{x+4}{3x-5}$.
Solution: First we drop the function notation and write y instead of $f(x)$. Then we solve for x and finally, swap x and y.

$$\begin{aligned} y &= \frac{x+4}{3x-5} && \text{multiply by } 3x-5 \\ y(3x-5) &= x+4 && \text{distribute} \\ 3xy-5y &= x+4 && \text{add } 5y, \text{ subtract } x \\ 3xy-x &= 5y+4 && \text{factor out } x \\ x(3y-1) &= 5y+4 && \text{divide by } 3y-1 \\ x &= \frac{5y+4}{3y-1} && \text{swap } x \text{ and } y \quad y = \frac{5x+4}{3x-1} \qquad f^{-1}(x) = \frac{5x+4}{3x-1} \end{aligned}$$

d) $f(x) = \log_5(2x-1)$
Solution: First we drop the function notation and write y instead of $f(x)$. Then we solve for x and finally, swap x and y.

$$\begin{aligned} y &= \log_5(2x-1) && \text{re-write it as an exponential statement} \\ 5^y &= 2x-1 && \text{add 1} \\ 5^y+1 &= 2x && \text{divide by 2} \\ \frac{5^y+1}{2} &= x \quad \text{swap } x \text{ and } y && y = \frac{1}{2}(5^x+1) \qquad f^{-1}(x) = \frac{1}{2}(5^x+1) \end{aligned}$$

e) $f(x) = e^{5x-1}$
Solution: First we drop the function notation and write y instead of $f(x)$. Then we solve for x and finally, swap x and y.

$$\begin{aligned} y &= e^{5x-1} && \text{take the natural logarithm of both sides} \\ \ln y &= \ln\left(e^{5x-1}\right) && \ln\left(e^{5x-1}\right) = 5x-1 \\ \ln y &= 5x-1 && \text{add 1} \\ \ln y+1 &= 5x && \text{divide by 5} \\ \frac{\ln y+1}{5} &= x \quad \text{swap } x \text{ and } y && y = \frac{1}{5}(\ln x+1) \qquad f^{-1}(x) = \frac{1}{5}(\ln x+1) \end{aligned}$$

2. Prove that the inverse of a linear function is also linear and the two slopes are reciporcals of each other.
Solution: Let $f(x) = mx+b$. If $m=0$, the function is very badly not on-to-one and the inverse does not exist. If $m \neq 0$, then the inverse is

$$\begin{aligned} y &= mx+b && \text{solve for } x \\ y-b &= mx && m \neq 0 \\ \frac{y-b}{m} &= x && \Longrightarrow \qquad x = \frac{1}{m}y - \frac{b}{m} \quad \Longrightarrow f^{-1}(x) = \frac{1}{m}x - \frac{b}{m} \end{aligned}$$

The inverse is a line with slope $\dfrac{1}{m}$.

