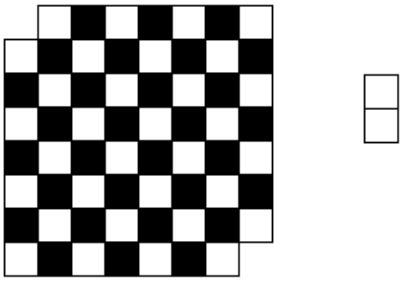
*These problems are mostly from my high school math teacher,* ***Prof. Lajos Posa*** *who is a well known mathematician/math teacher in Budapest. It is difficult to find words that describe his talent and dedication. So I just let the problems speak for themselves.*

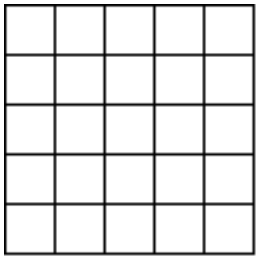
1. Two mathematicians are having a conversation. Mathematician A asks B about his kids. B answers: "I have three children, the product of their ages is 36." A says: "I still don't know how old your children are." Then B tells A the sum of his three kids' ages. A answers: "I still don't know how old they are. Then B adds: "The youngest one has red hair." Now A knows how old the kids are. Do you?

2. We are at a cross road. One road leads into a dangerous swamp, the other road leads into a town. There is a pair of identical twins on the crossing. We know that one twin always tells the truth, the other twin always lies. We are allowed to ask only one question from only one brother. Is there a way to find out which road leads to town?

3. Consider a chess board with two corners missing, as indicated on the picture below. We also have 31 pieces of domino, each of them can cover exactly 2 fields on the chess board. Is it possible to cover the chessboard with the domino pieces?



4. There is a 5x5 board as the picture below shows. So happens, on each one of the fields there is a ladybug sitting. Suddenly, each decides to move to a neighboring field. (Two fields are neighbors if they have an edge in common.) Is it possible that after each have moved there is again exactly one lady bug sitting on each field?



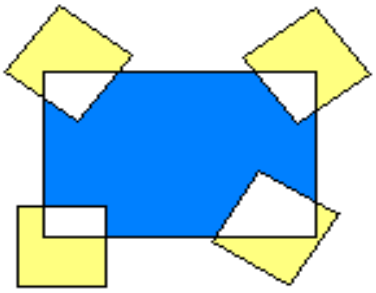
5. A king has his birthday. So he decides to let go some of his prisoners. He actually has 100 prisoners at the moment. They are each in a separate cell, numbered from 1 to 100. Well, he is a high-tech king. He can close or open any prison door by a single click on the cell's number on his royal laptop. When he clicks at a locked door, it opens. When he clicks at an open door, it locks. At the beginning, every door is locked. First the king clicks on every number from 1 to 100 (therefore opening every door). Then he clicks on every second number from 1 to 100, (i.e.2, 4, 6, 8, 10, . . . ). Then he clicks on every third number.(i.e. 3, 6, 9, 12, . . . ) Now he is opening some doors, locking others. Then he clicks on every fourth number. (i.e. 4, 8, 12, 16, . . . .) Then on every fifth.... And so on, every sixth, every seventh, etc. Until every 100th; finally, he only clicks on the number 100. Then he orders that the prisoners that find their door open may go free. Who gets to go and who has to stay?

6. Mr and Mrs Brown are having a party. They invited three other married couples, so there are eight people present. When greeting each other, some people shake hands with some people. Of course, nobody shakes hands with his/her spouse. When Mr Brown asks the other seven people: "How many people did you shake hands with?", he receives seven different answers. How many hands did Mrs. Brown shake?

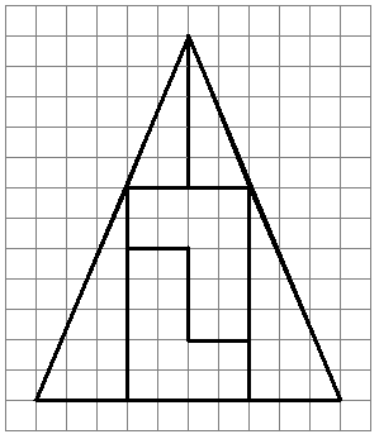
7. Mind-reading. Instructions:
   1.) Think of a 5-6 digit number, that has at least two different digits in it. My example is 803225.
   2.) Create a second number by rearranging the digits of the previous number. My example is 320258.
   3.) Subtract the smaller number from the larger number. My example is $803225 - 320258 = 482\,967$
   4.) Cross out any non-zero digit of the difference. My example is $482\,9\not{6}7$.
   5.) Announce the number you obtain by omitting the crossed out digit. My example is 482 97.

   The mindreader can tell what digit was crossed out. How?

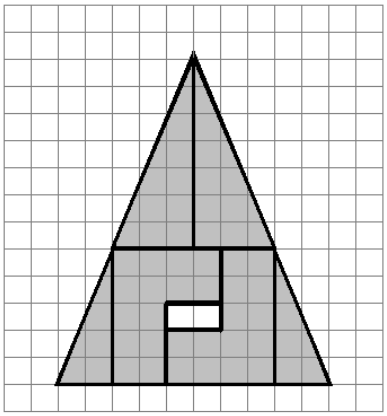
8. The picture below shows a rectangle (the sides' length are 2 and 3 unit long) and four identical squares (all four sides are 1 unit long). Determine which area is greater: the yellow or the blue?



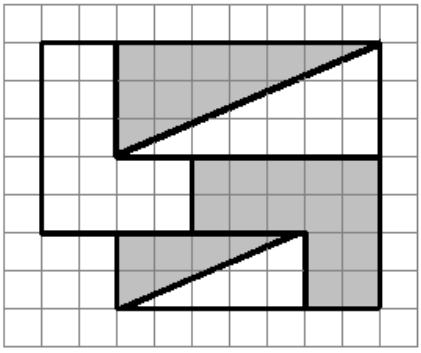
9. (Paul Curry) We cut the figure below out of paper. The area of the triangle is $A = \dfrac{10 \cdot 12}{2} = 60$ unit$^2$.



Suppose we color the other side of the paper, turn the pieces upside down, and rearrange them to obtain the figure shown below.



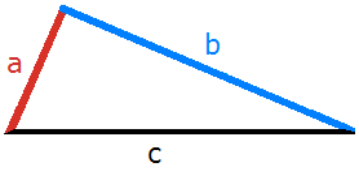
Now the area appears to be 58 unit$^2$ since there is a 2 unit$^2$ large gap. To make matters worse, we can again rearrange the pieces, turning some of them back to the original side.



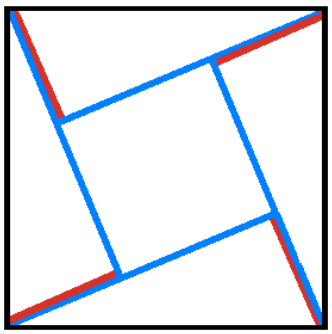
The area of this figure is $7\,(9) - 4 = 59$ unit$^2$.
Logic tells us that areas of figures do not change if we rearrange them or turn them on their other side. So, what is wrong with these pictures, and how much is really the are of these shapes, 58 unit$^2$, 59 unit$^2$, or 60 unit$^2$?

10. A bus makes a roundtrip between towns A and B. From A to B, the bus completes the trip with an average speed of 60 $\frac{\text{mi}}{\text{hr}}$. From B to A, it travels with an average velocity of 40 $\frac{\text{mi}}{\text{hr}}$. What is the average speed of the bus for the entire roundtrip? (Hint: it is NOT the average of 40 and 60)

11. There is a lot of different proofs of the Pythagorean Theorem. This is one I really like. Consider the right triangle shown on the picture below.

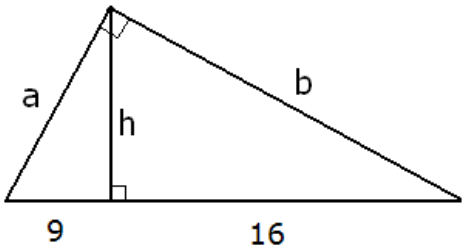



We used four identical right triangles to create the figure shown on the picture below.

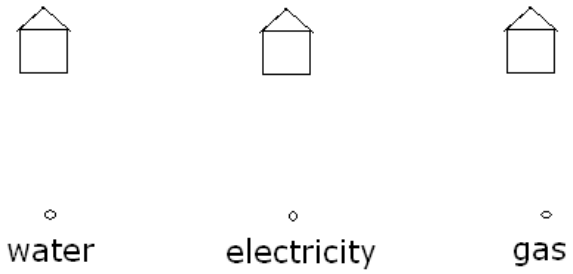


i) Find the area of the big square in terms of $c$.
ii) Find the area of the small rectangle in the middle in terms of $a$ and $b$.
iii) Find the area of the big square as the sum of 5 areas: four triangles and the small rectangle in the middle.

12. Find the value of $a$, $b$, and $h$, based on the picture below.

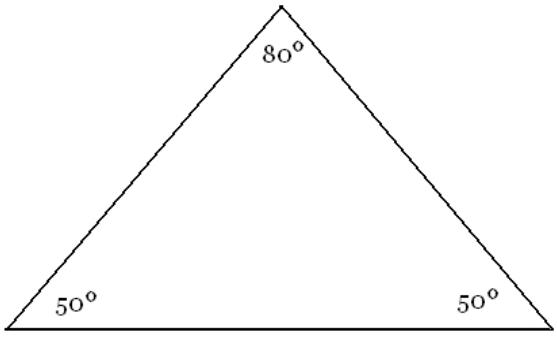


13. (This is a well known topology problem, and I heard it from my high school student, Eric.) Connect three buildings with the three utilities shown on the picture below. Each building has to be connected to all three utilities, in two dimensions, and no two utility pipes can cross each other.




14. There is a very difficult track in a mountain area, where the world record of driving a lap is 15 miles per hour average velocity. A race car driver announces that he intends to beat this record and run the course with an average velocity of 30 miles per hour. As he is driving, his progress is monitored. At the half of the track, his average velocity was 15 miles per hour. In spite of this, he arrives at the finish just a few seconds too late to have an average velocity of 30 miles per hour. His funeral was three days later. How did he die?

15. (From my student, Ramon Gozales.) The triangle below has angles 50°, 80°, and 80° as shown on the picture below.




We measure up 10° and 30° from the base as shown on the picture below. Find the measure of angle shown on the picture.