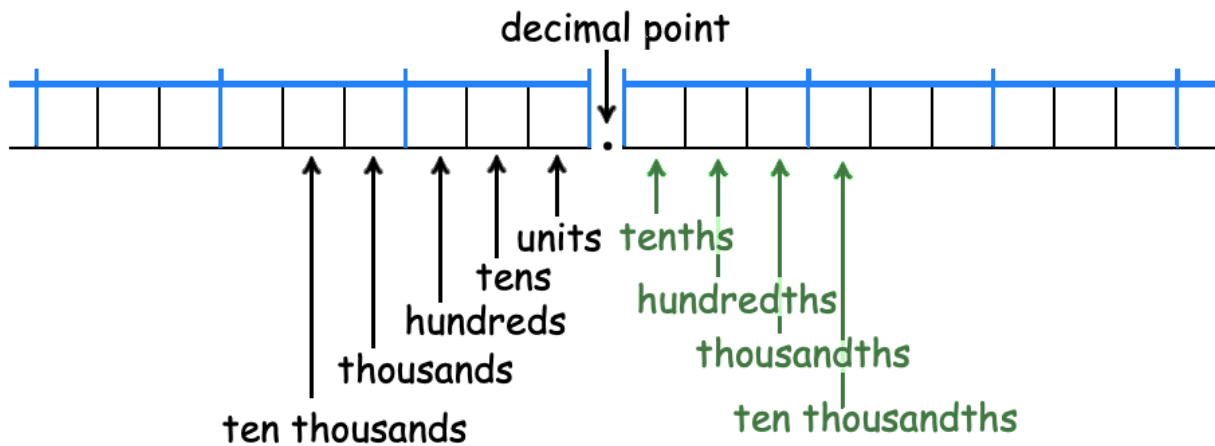


Decimals are standardized fractions with denominators 10, 100, 1000, etc. We express decimals by extending our system using place values and digits. The numerators are the digits, and the denominators are indicated by the place values.



1 is pronounced one

10 is pronounced ten

100 is pronounced hundred

1000 is pronounced one thousand

10 000 is pronounced ten thousand

100 000 is pronounced hundred thousand

1 000 000 is pronounced one million

0.1 is pronounced one-tenth and means $\frac{1}{10}$

0.01 is pronounced one-hundredth and means $\frac{1}{100}$

0.001 is pronounced one-thousandth and means $\frac{1}{1000}$

0.0001 is pronounced one ten-thousandth and means $\frac{1}{10\,000}$

0.000 01 is pronounced one hundred-thousandth and means $\frac{1}{100\,000}$

0.000 001 is pronounced one-millionth and means $\frac{1}{1\,000\,000}$

Example 1. Consider the decimal 35.702.

- a) Express the number using words. b) Re-write the number as a fraction of two integers.

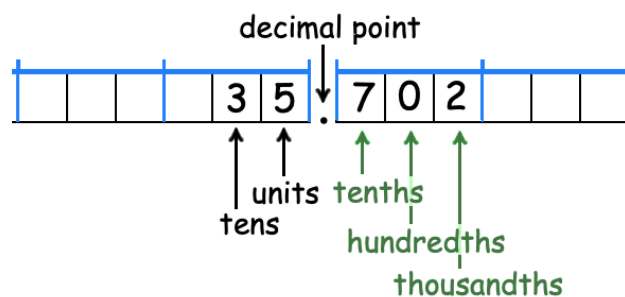
Solution: a) We can always read out a number following the steps shown below.

Step 1. Read out the number until the decimal point.

Step 2. For the decimal point, we say "and"

Step 3. After the decimal point, we read out the number as if it was a regular decimal with regular place values.

Step 4. We name the place value of the last digit.



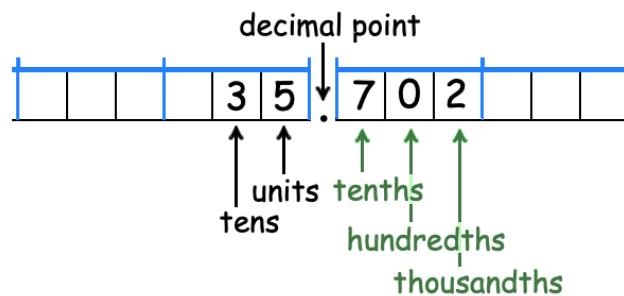
Step 1	Step 2	Step 3	Step 4
thirty-five	and	seven hundred two	thousandths

So the decimal is pronounced thirty-five and seven hundred two thousandths.

- b) If we can read out the number, we have automatically converted it into a mixed number.

thirty-five and seven hundred two one-thousandths can be written as $35 \frac{702}{1000}$ or $35 + \frac{702}{1000}$

Note that we get the same correct result if we interpret the decimal digit by digit and then add the fractions.



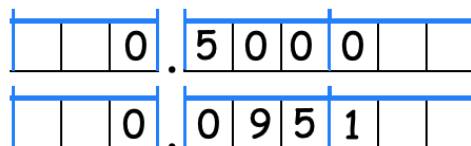
$$35 + \frac{7}{10} + \frac{0}{100} + \frac{2}{1000} = 35 + \frac{700 + 0 + 2}{1000} = 35 + \frac{702}{1000}$$

Example 2. Compare the decimals 0.5 and 0.0951

Solution: The number 951 is greater than 5 but that would be the incorrect comparison, because we need to consider the place values. We can re-write both numbers as fractions and compare them.

0.5 can be read as zero and five tenths, or $\frac{5}{10}$. The other decimal, 0.0951 can be read as zero and nine hundred fifty-one ten-thousandths, or as $\frac{951}{10000}$. To compare the decimals, we need to bring them to the common denominator: Compare $\frac{5000}{10000}$ and $\frac{951}{10000}$. 5000 is greater than 951 and so 0.5 is greater than 0.0951.

We can compare decimals in their decimal form as well, but we need to consider the place values. We can do that by making both numbers 'equally long':



Then we are comparing 5000 and 951. Even though 9 is greater than 5, 5000 is greater than 900.

Rounding Decimals

The basic principle and technique of rounding decimals are the same as it is with whole numbers. After identifying the place value to be rounded to, we round down if the next digit is 4 or less, and we round up if the next digit is five or more.

Example 3. Consider the number 628.4073096. Round this number to the nearest

- a) hundreds b) tens c) units d) one-tenths e) one-hundredths f) one-thousandths g) ten-thousandths

Solution: a) Our number, 628.4073096 is between 600 and 700. To correctly round, we need to figure out which one is nearest to our number. After the hundreds digit we see the digit 2. This digit is four or less, there fore, we round down and the answer is 600.

b) Our number, 628.4073096 is between 620 and 630. To correctly round, we need to figure out which one is nearest to our number. After the tens digit we see the digit 8. This digit is five or more, therefore we round up and the answer is 630. Indeed, 628.4073096 is closer to 630 than to 620.

c) Our number, 628.4073096 is between 628 and 629. To correctly round, we need to figure out which one is nearest to our number. After the units digit we see the digit 4. This digit is four or less, there fore, we round down and the answer is 628. Indeed, 628.4073096 is closer to 628 than to 629.

d) Our number, 628.4073096 is between 628.4 and 628, 5. To correctly round, we need to figure out which one is nearest to our number. After the one-tenths digit we see the digit 0. This digit is four or less, therefore, we round down and the answer is 628.4 Indeed, 628.4073096 is closer to 628.4 than to 628.5.

e) Our number, 628.4073096 is between 628.40 and 628, 41. To correctly round, we need to figure out which one is nearest to our number. After the one-hundredths digit we see the digit 7. This digit is five or more, there fore, we round up and the answer is 628.41 Indeed, 628.4073096 is closer to 628.41 than to 628.40.

f) Our number, 628.4073096 is between 628.407 and 628, 408. To correctly round, we need to figure out which one is nearest to our number. After the one-thousandths digit we see the digit 3. This digit is four or less, there fore, we round down and the answer is 628.407 Indeed, 628.4073096 is closer to 628.407 than to 628.408.

g) Our number, 628.4073096 is between 628.4073 and 628, 4074. To correctly round, we need to figure out which one is nearest to our number. After the ten–thousandths digit we see the digit 0. This digit is four or less, there fore, we round down and the answer is 628.4073 Indeed, 628.4073096 is closer to 628.4073 than to 628.4074.

1 Operations on Decimals

Part 1. Addition and Subtraction.

Addition and subtraction of decimals happen the same way as with whole numbers. In order to add the values of the corresponding placeholders, align the numbers by the decimal point. This is the same as adding fractions with common denominators.

Example 1. Perform the given additions.

a) $8.32 + 1.02$ b) $8.32 + 10.2$

Solution: We align the numbers by the decimal point. The sum will have the decimal point at the same place as the numbers to be added.

a)

$$\begin{array}{r} 8.32 \\ + 1.02 \\ \hline 9.34 \end{array}$$

b)

$$\begin{array}{r} 8.32 \\ + 10.2 \\ \hline 18.52 \end{array}$$

Note that aligning place values is the same as adding fractions with common denominators. For part a), we can re-write the numbers as fractions

$$8.32 = 8 + \frac{3}{10} + \frac{2}{100} \quad \text{and} \quad 1.02 = 1 + \frac{0}{10} + \frac{2}{100}$$

Adding the first column is $8 + 1 = 9$, adding the second column is $\frac{3}{10} + \frac{0}{10} = \frac{3}{10}$, and the third column is $\frac{2}{100} + \frac{2}{100} = \frac{4}{100}$.

The process is the same if digits add up to more than ten, and we carry over.

Example 2. Perform the addition $37.94 + 85.27$

Solution: We align the numbers by the decimal point. The sum will have the decimal point at the same place as the numbers to be added.

$$\begin{array}{r} 37.94 \\ + 85.27 \\ \hline 123.21 \end{array}$$

In case of subtraction too, the procedure remains the same as with whole numbers. We align the place values by the decimal point.

Example 3. Perform the subtraction $74.26 - 28.57$

Solution: We align the numbers by the decimal point. The difference will have the decimal point at the same place as the numbers to be subtracted.

$$\begin{array}{r} 74.26 \\ - 28.57 \\ \hline 45.69 \end{array}$$

Part 2. Multiplying and dividing decimals by 10-powers.

Let us recall that the decimal 0.037 is the same as the fraction $\frac{37}{1000}$. Let us see what happens when we multiply this decimal by ten-powers.

$$\frac{37}{1000} \cdot 10 = \frac{370}{1000} = \frac{37}{100} \quad \text{Therefore, } 0.037 \cdot 10 = 0.37$$

$$\frac{37}{1000} \cdot 100 = \frac{3700}{1000} = \frac{37}{10} = 3\frac{7}{10} \quad \text{Therefore, } 0.037 \cdot 100 = 3.7$$

$$\frac{37}{1000} \cdot 1000 = \frac{37000}{1000} = 37 \quad \text{Therefore, } 0.037 \cdot 1000 = 37$$

$$\frac{37}{1000} \cdot 10\,000 = \frac{370\,000}{1000} = 370 \quad \text{Therefore, } 0.037 \cdot 10\,000 = 370$$

To multiply a decimal by a 10–power, we simply move the decimal point to the right. The number of decimal places to be moved is the same as the number of zeroes after the 1 in the 10–power. So, to multiply by 1000, we move the decimal point three places to the right. If the decimal point reaches the end of the number, we add zeroes to the end.

Division by 10–powers will work similarly, only we will move the decimal to the left.



Practice Problems

1. Write each of the given numbers using digits.
 - a) twenty-eight and three thousand two hundred thirteen millionths
 - b) eighty thousand four and four hundred-thousandths
 - c) forty-two and four thousand twenty-five ten-thousandths
2. Write each of the given decimals using letters.
 - a) 0.25 b) 21.8005 c) 2.0021 d) 100.000301 e) 195.61023
3. Write each of the given decimals as a reduced fraction.
 - a) 0.5 b) 0.32 c) 0.025
4. Round 3802.75428 to the
 - a) nearest thousands d) nearest units g) nearest one-thousandths
 - b) nearest hundreds e) nearest one-tenths h) nearest ten-thousandths
 - c) nearest tens f) nearest one-hundredths



Answers

Practice Problems

1. a) 28.003213 b) 80 004.00004 c) 42.4025
1. a) zero and twenty-five hundredths b) twenty-one and eight thousand five ten-thousandths
c) two and twenty-one ten-thousandths d) one hundred and three hundred one one-millionths
e) one hundred ninety-five and sixty-one thousand, twenty-three hundred-thousandths
2. a) $\frac{1}{2}$ b) $\frac{8}{25}$ c) $\frac{1}{40}$
3. a) 4000 b) 3800 c) 3800 d) 3803 e) 3802.8 f) 3802.75 g) 3802.754 h) 3802.7543