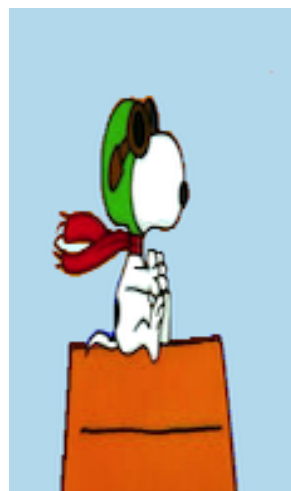


Before the digital age, pictures were developed into photographs. A standard photograph would be a 3 by 5, meaning that the horizontal longer side is 5 inches long, and the vertical side is 3 inches long.



Suppose we want to enlarge a standard photograph. If we just increase one side but not the other, or just randomly increase the two sides, the image becomes distorted. The only way to enlarge the picture and preserve the image on it is by multiplying both sides by the same number. For example, we can double both sides: the horizontal side would become 10 inches and the vertical side would become 6 inches long. We can 'triple' the image, for sides 15 inches and 9 inches long. In either case, the ratio of the horizontal side to the vertical side remains the same.



Definition: A **ratio** is a comparison of two quantities by division.

In this case, the ratio of the vertical side to the horizontal side should remain the same. 3 to 5, or 6 to 10, or 9 to 15. A ratio can be written in several different ways:

$$3 \text{ to } 5 \quad \text{or} \quad 3 : 5 \quad \text{or} \quad \frac{3}{5} \quad \text{or} \quad 3 \div 5.$$

Example 1. The ratio 2 to 7 can be written as $\frac{2}{7}$ or $2 : 7$ or $2 \div 7$.

Example 2. The ratio 20 to 15 can be written as $\frac{20}{15}$. We can reduce this fraction to lowest terms as $\frac{4}{3}$. Therefore, the ratio 20 to 15 is the same as 4 to 3.

Example 3. The computer lab at our school had 42 PCs and 35 Macintosh. Express and simplify each of the given ratios.
a) the ratio of PCs to Macs b) the ratio of Macs to PCs

Solution: a) The ratio of PCs to Macs is 42 to 35. As a fraction, that is $\frac{42}{35}$. We can reduce this fraction and get $\frac{6}{5}$ or 6 to 5.
b) The ratio of Macs to PCs is 35 to 42. As a fraction, that is $\frac{35}{42}$. We can reduce this fraction and get $\frac{5}{6}$ or 5 to 6.

Example 4. Simplify the ratio $\frac{3}{7}$ to $\frac{4}{7}$.

Solution: We can write the ratio as a fraction and simplify.

$$\frac{\frac{3}{7}}{\frac{4}{7}} \quad \text{to divide is to multiply by the reciprocal.}$$

$$\frac{\frac{3}{7}}{\frac{4}{7}} = \frac{3}{7} \cdot \frac{7}{4} = \frac{3}{4} \text{ or 3 to 4.}$$

Example 5. Simplify the ratio $3\frac{1}{3}$ to $2\frac{1}{2}$.

Solution: We can write the ratio as a fraction and simplify. Before we do that, we need to re-write the mixed numbers as improper fractions.

$$\frac{3\frac{1}{3}}{2\frac{1}{2}} \quad \text{to divide is to multiply by the reciprocal.}$$

$$\frac{\frac{10}{3}}{\frac{5}{2}} = \frac{10}{3} \cdot \frac{2}{5} = \frac{4}{3} \text{ or 4 to 3.}$$

Example 6. Express and simplify the ratio $4\frac{1}{6}$ hours to 100 minutes.

Solution: We will use the fact that 1 hour is 60 minutes and convert from hours to minutes. We will use a conversion factor.

$$4\frac{1}{6} \text{ hour} = \frac{25 \cancel{\text{hr}}}{6} \cdot \frac{60 \text{ min}}{\cancel{\text{hr}}} = 250 \text{ minutes}$$

Then the ratio as a fraction is $\frac{250 \text{ min}}{100 \text{ min}} = \frac{25}{10} = \frac{5}{2}$. Thus the ratio is 5 to 2.

Example 7. Express and simplify the ratio $1\frac{1}{4}$ feet to 10 inches.

Solution: We will use the fact that 1 feet is 12 inches and convert from feet to inches. We will use a conversion factor.

$$1\frac{1}{4} \text{ ft} = \frac{5 \text{ ft}}{4} = \frac{5 \cancel{\text{ft}}}{4} \cdot \frac{12 \text{ in}}{\cancel{1 \text{ ft}}} = 15 \text{ in}$$

Then the ratio as a fraction is $\frac{15 \text{ in}}{10 \text{ in}} = \frac{3}{2}$. Thus, the reduced ratio is $\frac{3}{2}$.

In many, many, many aspects of our life, quantities related to each other have naturally constant ratios. For example, the quantity of a product we purchase and the amount of money we pay for them. If 2 pounds of rice costs 3 dollars, then the ratio of the money we pay and the weight of the rice is constant.

quantity	money we pay
2 pounds	3 dollars
4 pounds	6 dollars
20 pounds	30 dollars

We call that constant ratio the unit price: $\frac{3 \text{ dollar}}{2 \text{ pounds}} = 1\frac{1}{2}$ dollars per pound

Example 8. According to a survey, four out of five people when asked, claims to have a hobby.

- If we asked 1000 people, how many claimed to have a hobby?
- If 1000 people claimed to have a hobby, how many people were asked as part of the survey?

Solution: a) The ratio of people with a hobby and the people asked is constant, 4 to 5. Let us denote the number of people with a hobby by x . Then the ratio can be expressed in two different ways, giving us an equation.

$$\frac{4}{5} = \frac{x}{1000}$$

There are several ways to solve this equation. We will treat it as a one-step equation where x was divided by 1000. So we isolate x by multiplying both sides by 1000.

$$x = \frac{4}{5} \cdot 1000 = \frac{4000}{5} = 800$$

Therefore, 800 people claimed to have a hobby out of the 1000 asked. Indeed, the ratio 800 to 1000 is the same ratio as 4 to 5.

- The ratio of the number of people with a hobby and of the people asked is constant, 4 to 5. Let us denote the number of people asked by y . Then the ratio can be expressed in two different ways, giving us an equation.

$$\frac{4}{5} = \frac{1000}{y}$$

If $A = B$, then $\frac{1}{A} = \frac{1}{B}$. One nice thing about ratios is that if we don't like the fact that the unknown is in the denominator, we can take the reciprocal of both sides. That would be the same as looking at the ratio of the number of people asked to the number of people with hobbies.

$$\frac{5}{4} = \frac{y}{1000}$$

So we isolate y by multiplying both sides by 1000.

$$y = \frac{5}{4} \cdot 1000 = \frac{5000}{4} = 1250$$

Therefore, 1250 people were asked. Indeed, the ratio 1000 to 1250 is the same ratio as 4 to 5.

Example 9. We have a recipe for cookies. The quantities described in the recipe are for 6 servings, but we want to bake for only four people. The recipe calls for $1\frac{2}{3}$ cups of flour and $\frac{3}{4}$ cups of sugar. How much flour and sugar should we use?

Solution: The constant ratio in this situation is the amount used to the number of people. These ratios can be expressed as

for flour: $\frac{1\frac{2}{3} \text{ cups}}{6 \text{ people}}$ or $\frac{1\frac{2}{3}}{6}$ and $\frac{\frac{3}{4} \text{ cups}}{6 \text{ people}}$ or $\frac{\frac{3}{4}}{6}$ for sugar.

Let us denote the amount of flour we need for 4 people by x . The constant ratio then can be expressed in two different ways, giving us an equation in x .

$$\frac{1\frac{2}{3} \text{ cups}}{6} = \frac{x}{4}$$

We can solve this equation by multiplying both sides by 4.

$$\frac{1\frac{2}{3} \text{ cups}}{6} \cdot 4 = x$$

So we get that

$$x = \frac{1\frac{2}{3} \text{ cups}}{6} \cdot 4 = \frac{4 \cdot 1\frac{2}{3} \text{ cups}}{6} = \frac{4 \cdot \frac{5}{3}}{6} \text{ cups} = \frac{\frac{20}{3}}{\frac{6}{1}} \text{ cups} = \frac{20}{3} \cdot \frac{1}{6} \text{ cups} = \frac{10}{9} \text{ cup} = 1\frac{1}{9} \text{ cups}$$

Similarly, if we denote the amount of flour we need for 4 people by y . The constant ratio then can be expressed in two different ways, giving us an equation in y .

$$\frac{\frac{3}{4} \text{ cups}}{6} = \frac{y}{4}$$

We can solve this equation by multiplying both sides by 4.

$$\frac{\frac{3}{4} \text{ cups}}{6} \cdot 4 = y$$

$$\text{So we get that } y = \frac{\frac{3}{4} \text{ cups}}{6} \cdot 4 = \frac{4 \cdot \frac{3}{4} \text{ cups}}{6} = \frac{3}{6} \text{ cups} = \frac{1}{2} \text{ cup}$$



Practice Problems

- Re-write each of the given ratios as a reduced fraction.
 - 8 to 18
 - 18 to 8
 - 5 to 15
 - $\frac{2}{3}$ to $\frac{5}{6}$
 - 60% to $\frac{3}{4}$
 - 14 inches to one foot
 - 300 minutes to three hours
 - 30 nickels to 5 dollars
- The party was a great success. There were 120 women and 100 men in attendance. Find each of the given ratios.
 - the number of men to the number of women
 - the number of men to the number of guests
 - the number of women to the number of guests
- There is no student at our school who takes both French and Spanish. For every four students studying French at our school, there are 14 students who take Spanish
 - If 378 students are taking French, how many are taking Spanish?
 - If 378 students are taking Spanish, how many are taking French?
 - If all together 378 students are taking French or Spanish, how many are taking French? And Spanish? (Hint: if 4 students take French and 14 take Spanish, then 4 out 18 students are taking French.)
- On our map, a distance of 3 inches represents 500 miles.
 - What distance is denoted by $4\frac{1}{5}$ inches?
 - If two cities are 125 miles from each other, what is their distance on the map?



Answers

- $\frac{4}{9}$
 - $\frac{9}{4}$
 - $\frac{1}{3}$
 - $\frac{4}{5}$
 - $\frac{4}{5}$
 - $\frac{7}{6}$
 - $\frac{5}{3}$
 - $\frac{3}{10}$
- $\frac{5}{6}$
 - $\frac{5}{11}$
 - $\frac{6}{11}$
- 1323
 - 108
 - 84 take French and 294 take Spanish
- 700 miles
 - $\frac{3}{4}$ inch