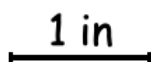
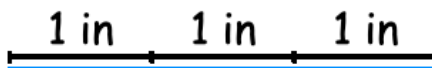


Perimeter is a measure of length. The length of an object, measured in inches, is the number of 1-inch line segments we need to cover our object.

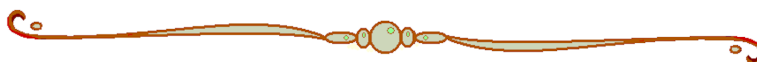


Unit inch

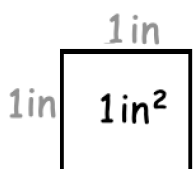


This blue line is 3 inches long.

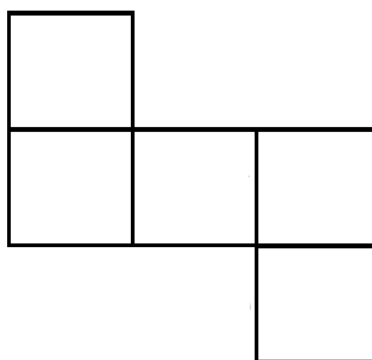
Perimeter, and all lengths are measured in meters (m), inches (in), feet (ft), miles (mi), etc.



Area is a measure of surface. Area of an object, measured in square-inches, is the number of 1-inch by 1-inch squares we need to cover our object.



1 square-inch

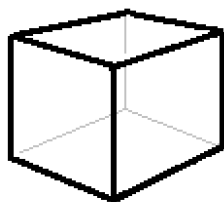


This area is 5 square-inches.

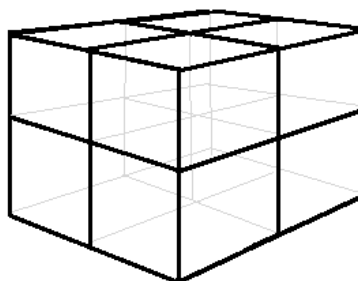
Area is measured in square-meters (m^2), square-inches (in^2), square-feet (ft^2), square-miles (mi^2), etc.



Volume is a measure of space. The volume of an object, measured in cubic inches, is the number of 1 inch by 1 inch by 1 inch cubes we need to build a replica of our object. Another way to imagine this is to make an ice-sculpture replica of our object, melt it and then freeze it to unit cubes and count.



1 cubic inch



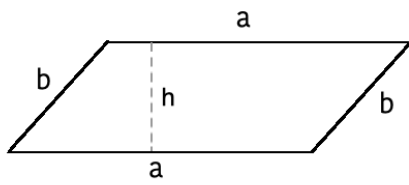
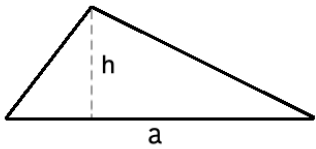
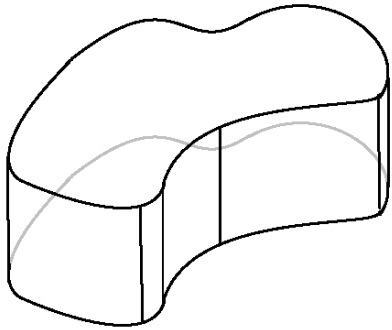
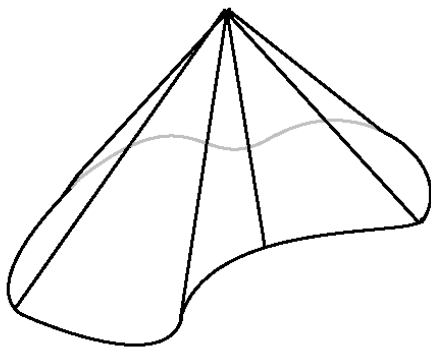
The volume of this object is 8 cubic-inches.

Volumes are measured in cubic meters (m^3), cubic inches (in^3), cubic feet (ft^3), cubic miles (mi^3), etc.

Another way of thinking of all this is that length is one-dimensional, part of a line while area, part of a plane is two-dimensional, and volume, part of space is three-dimensional. The space we live in can be described as three-dimensional.

This idea might help us to develop an easy method of remembering volume formulas. Let us first develop the concept of "solid objects" and "pointy object". A solid would start in a shape and end in the same shape. A pointy object would start in a shape and end in a point.

For example, in the plane (2-dimensions), a solid could be a parallelogram while a pointy object would be a triangle.

2-dimensional solid Area is base times height $A = ah$ 	2-dimensional pointy Area is base times height over 2 $A = \frac{ah}{2}$ 
3-dimensional solid Volume is base area times height $V = Bh$ where B is the area of the base 	3-dimensional pointy Volume is base area times height over 3 $V = \frac{Bh}{3}$ where B is the area of the base 

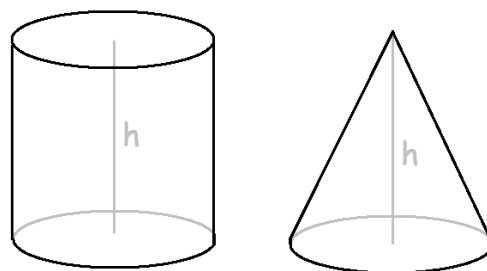
Maybe we cannot visualize objects in 4 dimension, but their 'volume' formula is probably $\frac{Vh}{4}$, i.e. base-volume times the height, over four. Either way, the point here is that using this general rule is easier than memorizing many volume formulas.

Example 1. The solid shown on the picture is called a cylinder. The pointy object is called a cone. Find the volume of each if the base is a circle with area 30in^2 and the height is 12in .

Solution: We do not yet know the formula for the area of the circle, but this area is given. We apply our new volume formulas.

For the cylinder, $V = Bh = 30\text{in}^2 \cdot 12\text{in} = 360\text{in}^3$

For the cone, $V = \frac{Bh}{3} = \frac{30\text{in}^2 \cdot 12\text{in}}{3} = 120\text{in}^3$.



Example 2. The solid shown on the picture is a rectangular prism. The pointy object is called a rectangular pyramid. Find the volume of each objects.

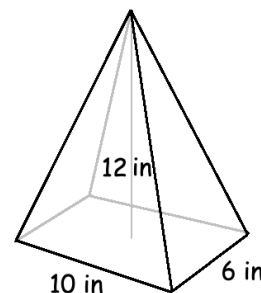
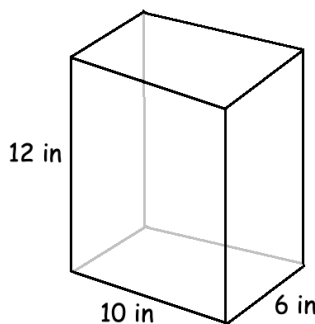
Solution: We apply our new volume formulas.

In both cases, the base is a rectangle with sides 6in and 10in.

The area of the base is $B = 6\text{in} \cdot 10\text{in} = 60\text{in}^2$. The height is, in both cases, 12in.

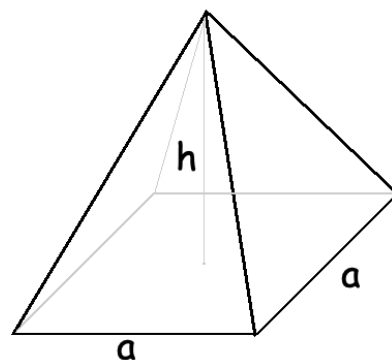
For the rectangular prism, $V = Bh = 60\text{in}^2 \cdot 12\text{in} = 720\text{in}^3$

For the pyramid, $V = \frac{Bh}{3} = \frac{60\text{in}^2 \cdot 12\text{in}}{3} = 240\text{in}^3$.



Practice Problems

1. What is the volume of a square with edges 5ft long?
2. Find the volume of the square based pyramid shown on the picture, where $a = 4\text{cm}$ and $h = 6\text{cm}$.
3. Suppose that the volume of a rectangular prism is 24m^3 . How many such prisms are possible if all three sides are whole numbers?





Answers

1. $V = 125\text{ft}^3$ 2. 32cm^3
2. 6 possibilities: $(1, 1, 24)$, $(1, 2, 12)$, $(1, 3, 8)$, $(1, 4, 6)$, $(2, 2, 6)$, $(2, 3, 4)$

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