

Part 1 - More Fractions

We will further study solving linear equations. Let us first recall a few definitions.

Definition: An **equation** is a statement in which two expressions (algebraic or numeric) are connected with an equal sign. A **solution** of an equation is a number (or an ordered set of numbers) that, when substituted into the variable(s) in the equation, makes the statement of equality of the equation true. To **solve an equation** is to find *all* solutions of it. The set of all solutions is also called the solution set.

Example 1. Solve each of the given equations.

$$\text{a) } \frac{2x-5}{3} - \frac{x-2}{5} = x-5 \qquad \text{b) } \frac{2}{3}(x-1) - \frac{1}{2}\left(x + \frac{3}{5}\right) = -x + \frac{37}{10}$$

Solution: a) The main idea here is that we can clear denominators of fractions in equations by multiplying by a suitable number. As always, we will multiply both sides by a suitable number.

$\frac{2x-5}{3} - \frac{x-2}{5} = x-5$	we write everything as a fraction
$\frac{2x-5}{3} - \frac{x-2}{5} = \frac{x-5}{1}$	bring all three fractions to the common denominator
$\frac{5(2x-5)}{15} - \frac{3(x-2)}{15} = \frac{15(x-5)}{15}$	clear denominators by multiplying by 15
$5(2x-5) - 3(x-2) = 15(x-5)$	remove parentheses
$10x - 25 - 3x + 6 = 15x - 75$	combine like terms
$7x - 19 = 15x - 75$	subtract $7x$
$-19 = 8x - 75$	add 75
$56 = 8x$	divide by 8
$7 = x$	

We check: if $x = 7$, then

$$\text{LHS} = \frac{2 \cdot 7 - 5}{3} - \frac{7 - 2}{5} = \frac{14 - 5}{3} - \frac{5}{5} = \frac{9}{3} - 1 = 3 - 1 = 2 \quad \text{and} \quad \text{RHS} = 7 - 5 = 2$$

and so our solution, $\boxed{x = 7}$ is correct.

Note that it is faster if we multiply both sides by the common multiple without bringing the terms to the common denominator. Here is the recommended computation:

$$\begin{aligned} \frac{2x-5}{3} - \frac{x-2}{5} &= \frac{x-5}{1} && \text{multiply both sides by 15} \\ 5(2x-5) - 3(x-2) &= 15(x-5) \end{aligned}$$

Why is this step correct? Here is the breakdown:

$$\frac{2x-5}{3} - \frac{x-2}{5} = \frac{x-5}{1}$$

$$15 \left(\frac{2x-5}{3} - \frac{x-2}{5} \right) = 15 \left(\frac{x-5}{1} \right)$$

$$\frac{15}{1} \cdot \frac{2x-5}{3} - \frac{15}{1} \cdot \frac{x-2}{5} = \frac{15}{1} \cdot \frac{x-5}{1}$$

$$\frac{15(2x-5)}{3} - \frac{15(x-2)}{5} = \frac{15(x-5)}{1}$$

$$5(2x-5) - 3(x-2) = 15(x-5)$$

- b) There are several methods available to solve this equation. The method presented here is focusing on how similar this equation is to the previous example.

$$\frac{2}{3}(x-1) - \frac{1}{2} \left(x + \frac{3}{5} \right) = -x + \frac{37}{10}$$

$$x = 1x = \frac{5}{5}x \quad \text{and} \quad -x = -1x = -\frac{10}{10}x$$

$$\frac{2}{3}(x-1) - \frac{1}{2} \left(\frac{5x}{5} + \frac{3}{5} \right) = -\frac{10x}{10} + \frac{37}{10}$$

$$\frac{2}{3} \cdot \frac{x-1}{1} - \frac{1}{2} \cdot \frac{5x+3}{5} = \frac{-10x+37}{10}$$

$$\frac{2(x-1)}{3} - \frac{5x+3}{10} = \frac{-10x+37}{10}$$

bring all fractions to the common denominator

$$\frac{20(x-1)}{30} - \frac{3(5x+3)}{30} = \frac{3(-10x+37)}{30}$$

clear denominator by multiplying by 30

$$20(x-1) - 3(5x+3) = 3(-10x+37)$$

remove parentheses

$$20x - 20 - 15x - 9 = -30x + 111$$

combine like terms

$$5x - 29 = -30x + 111$$

add 30x

$$35x - 29 = 111$$

add 29

$$35x = 140$$

divide by 35

$$x = 4$$

We check: if $x = 4$, then

$$\text{LHS} = \frac{2}{3}(4-1) - \frac{1}{2} \left(4 + \frac{3}{5} \right) = \frac{2}{3} \cdot 3 - \frac{1}{2} \left(\frac{20}{5} + \frac{3}{5} \right) = 2 - \frac{1}{2} \cdot \frac{23}{5} = 2 - \frac{23}{10} = \frac{20}{10} - \frac{23}{10} = \frac{-3}{10}$$

$$\text{RHS} = -4 + \frac{37}{10} = -\frac{40}{10} + \frac{37}{10} = -\frac{3}{10}$$

and so our solution, $\boxed{x = 4}$ is correct.

Just as before, we can get from $\frac{2(x-1)}{3} - \frac{5x+3}{10} = \frac{-10x+37}{10}$ immediately to

$$2 \cdot 10(x-1) - 3(5x+3) = 3(-10x+37)$$

by multiplying both sides by 30 without bringing the fractions to lowest terms.

Example 2. Solve each of the given equations. Make sure to check your solutions.

a) $\frac{1}{2}m - 1 = \frac{5}{4}m - \frac{1}{4}$ b) $\frac{2}{3}x - 4 - \frac{1}{6}(x + 6) = \frac{1}{2}(x - 10)$

Solution:

a)	$\frac{1}{2}m - 1 = \frac{5}{4}m - \frac{1}{4}$	subtract $\frac{1}{2}m$	margin work: $\frac{5}{4} - \frac{1}{2} = \frac{5}{4} - \frac{2}{4} = \frac{3}{4}$
	$-1 = \frac{3}{4}m - \frac{1}{4}$	add $\frac{1}{4}$	$-1 + \frac{1}{4} = \frac{-4}{4} + \frac{1}{4} = -\frac{3}{4}$
	$-\frac{3}{4} = \frac{3}{4}m$	divide by $\frac{3}{4}$	$-\frac{3}{4} \div \frac{3}{4} = -\frac{3}{4} \cdot \frac{4}{3} = -1$
	$-1 = m$		

So the only solution of this equation is -1 . We check; if $m = -1$,

$$\text{LHS} = \frac{1}{2}(-1) - 1 = -\frac{1}{2} - 1 = -\frac{1}{2} - \frac{2}{2} = -\frac{3}{2} \text{ and}$$

$$\text{RHS} = \frac{5}{4}(-1) - \frac{1}{4} = -\frac{5}{4} - \frac{1}{4} = -\frac{6}{4} = -\frac{3}{2} \quad \implies \quad \text{LHS} = \text{RHS}$$

So our solution, $\boxed{m = -1}$ is correct.

b) We first eliminate the parentheses by applying the distributive law.

$\frac{2}{3}x - 4 - \frac{1}{6}(x + 6)$	$=$	$\frac{1}{2}(x - 10)$	eliminate parentheses	
$\frac{2}{3}x - 4 - \frac{1}{6}x - 1$	$=$	$\frac{1}{2}x - 5$	combine like terms	margin work: $\frac{2}{3} - \frac{1}{6} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2}$
$\frac{1}{2}x - 5$	$=$	$\frac{1}{2}x - 5$	subtract $\frac{1}{2}x$	
-5	$=$	-5		

Our last line indicates that this equation is an identity; all numbers are solutions of it.

Part 2 - Linear Equations after Multiplying Expressions

Now that we can multiply algebraic expressions, we can solve equations that include such products.

Example 3. Solve the given equation. Make sure to check your solution.

$$(2x-3)^2 - (x+1)(3x-5) = 11 - (x-1)(3-x)$$

Solution: We carefully expand the indicated products and combine like terms. Notice that even after we expanded $(x+1)(3x-5)$ and $(x-1)(3-x)$, we still need to keep them in parentheses because we are subtracting them. We will first work out the products.

$$\begin{aligned}(2x-3)^2 &= (2x-3)(2x-3) = 4x^2 - 6x - 6x + 9 = 4x^2 - 12x + 9 \\(x+1)(3x-5) &= 3x^2 - 5x + 3x - 5 = 3x^2 - 2x - 5 \\(x-1)(3-x) &= 3x - x^2 - 3 + x = -x^2 + 4x - 3\end{aligned}$$

We are now ready to begin solving the equation.

$$\begin{aligned}(2x-3)^2 - (x+1)(3x-5) &= 11 - (x-1)(3-x) \\4x^2 - 12x + 9 - (3x^2 - 2x - 5) &= 11 - (-x^2 + 4x - 3)\end{aligned}$$

to subtract is to add the opposite

$$\begin{aligned}4x^2 - 12x + 9 - 3x^2 + 2x + 5 &= 11 + x^2 - 4x + 3 && \text{combine like terms} \\x^2 - 10x + 14 &= x^2 - 4x + 14 && \text{subtract } x^2 \\-10x + 14 &= -4x + 14 && \text{add } 10x \\14 &= 6x + 14 && \text{subtract } 14 \\0 &= 6x && \text{divide by } 6 \\0 &= x\end{aligned}$$

We check: if $x = 0$, then

$$\begin{aligned}\text{LHS} &= (2 \cdot 0 - 3)^2 - (0 + 1)(3 \cdot 0 - 5) = (-3)^2 - 1(-5) = 9 + 5 = 14 \\ \text{RHS} &= 11 - (0 - 1)(3 - 0) = 11 - (-1)3 = 11 + 3 = 14\end{aligned}$$

and so our solution, $x = 0$ is correct.

Example 4. The area of a square would increase by 93 unit^2 if we increased each of its sides by 3 units. How long are the sides of the square?

Solution: Let us denote the side of the square by x . Then its area is x^2 . If we increase its sides by 3 units, the new side would be $x + 3$, and the new area $(x + 3)^2$. The equation would express the difference between the two areas.

$$\begin{aligned}(x + 3)^2 &= x^2 + 93 \\ x^2 + 6x + 9 &= x^2 + 93 && \text{subtract } x^2 \\ 6x + 9 &= 93 && \text{subtract 9} \\ 6x &= 84 && \text{divide by 6} \\ x &= 14\end{aligned}$$

Thus the square has sides 14 units long. We check: The area of the smaller square is $14^2 = 196 \text{ unit}^2$. If we increase the sides by 3 units, they would be 17 units. Then the area is $17^2 = 289 \text{ unit}^2$. The difference between the two areas is indeed 93 unit^2 , as $289 - 196 = 93$. Thus our solution is correct.



Sample Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x + 3 = 4x + 9$

5. $7(j - 5) + 9 = 2(-2j + 5) + 5j$

9. $\frac{2}{3}(x - 1) = \frac{3}{5}(x - 4) + 1$

2. $3w - 5 = 5(w + 1)$

6. $3(x - 5) - 5(x - 1) = -2x + 1$

10. $\frac{2}{3}(x - 7) = \frac{4}{5}(x + 1)$

3. $3y - 9 = -2y + 4$

7. $\frac{3 - x}{4} - \frac{10 - 3x}{5} = x + 2$

11. $\frac{x + 2}{4} - \frac{x - 3}{5} = 20 - x$

4. $4 - x = 3(x - 7)$

8. $\frac{3x + 17}{2} = x - 1 + \frac{x + 19}{2}$

12. $(x - 3)^2 - (2x - 5)(x + 1) = 5 - (x - 1)^2$

13. $(x + 1)^2 - (2x - 1)^2 + (3x)^2 = 6x(x - 2)$

14. $12 - (2p - 1)(p + 1) = -2(-p + 5)^2$

15. If we increase all sides of a square by 3 units, the area of the square increase by 75 units. How long are the sides of the square?



Practice Problems

Solve each of the following equations. Make sure to check your solutions.

1. $5x - 3 = x + 9$
2. $-x + 13 = 2x + 1$
3. $-2x + 4 = 5x - 10$
4. $5x - 7 = 6x + 8$
5. $8x - 1 = 3x + 19$
6. $-7x - 1 = 3x - 21$
7. $3(x - 4) = 2(x + 5)$
8. $4(5x + 1) = 6x + 4$
9. $a - 3 = 5(a - 1) - 2$
10. $3y - 2 = -2y + 18$
11. $8(x - 3) - 3(5 - 2x) = x$
12. $5(x - 1) - 3(x + 1) = 3x - 8$
13. $-2x - (3x - 1) = 2(5 - 3x)$
14. $3(x - 4) + 5(x + 8) = 2(x - 1)$
15. $5(x - 1) - 3(-x + 1) = -3 + 8x$
16. $\frac{3x - 1}{5} - \frac{7 - x}{3} = 2x + 6$
17. $\frac{3x - 1}{4} + \frac{8 - 4x}{3} = -3 - x$
18. $\frac{3x - 2}{5} + \frac{x + 4}{3} = \frac{14(x + 1)}{15}$
19. $\frac{3}{8}x + 1\frac{4}{5} = \frac{1}{4}x + 1\frac{3}{10}$
20. $\frac{2x + 1}{3} - \frac{3 - x}{2} = x - 2$
21. $\frac{2}{3}x - 1 = -\frac{2}{3}\left(x + \frac{1}{2}\right)$
22. $2(b + 1) - 5(b - 3) = 2(b - 7) + 1$
23. $3(2x - 1) - 5(2 - x) = 4(x - 1) + 5$
24. $3(2x - 7) - 2(5x + 2) = -5x - 30$
25. $3(x - 4) - 4(x - 3) = 3(x - 2) + 2(3 - x)$
26. $2x(3x - 1) - x(5x - 2) = (x - 1)^2$
27. $y^2 - (y - 1)^2 + (y - 2)^2 = (y - 3)(y - 5)$
28. $(3x)^2 - (x + 3)(5x - 3) = (5 - 2x)^2 - 16$
29. $(w + 4)(1 - 2w) = 3w - 2(w - 3)^2$
30. $(2x - 3)^2 - 3(x - 2)^2 = 10 - (x - 2)(7 - x)$
31. $(2 - w)^2 - (2w - 3)^2 + 7 = (w - 2)(5 - 3w)$
32. $3(a + 11) - a(8 - 3a) = 3(a - 2)^2$
33. $-5(2x - 1) - (4 - x)^2 = 3 - (x + 1)^2$
34. $5(-3 - x) - 3x(x - 2) = x - 3(x + 2)(x - 5)$
35. $2(-m - 2)^2 - (m - 2)^2 = 8m + (m + 2)^2$
36. $(3a - 5)(2 - a) - (2a - 1)(a + 3) = -5a^2 - 7$
37. $\frac{1}{2}(x - 3)^2 - \frac{1}{2}(x + 1)^2 = 4(x - 7)$
38. If we increase all sides of a square by 6 units, the square's area increase by 144 unit². How long are the sides of the square?



Answers

Discussion

a) contradiction b) conditional c) conditional d) identity

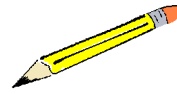
Sample Problems

1. -3 2. -5 3. $\frac{13}{5}$ 4. $\frac{25}{4}$ 5. 6 6. no solution 7. -5 8. identity, all numbers are solution 9. -11
10. -41 11. 18 12. 2 13. 0 14. 3 15. 11 units

Practice Problems

1. 3 2. 4 3. 2 4. -15 5. 4 6. 2 7. 22 8. 0 9. 1 10. 4 11. 3 12. 0 13. 9 14. -5
15. there is no solution 16. -8 17. -13 18. all real numbers are solution 19. -4 20. -5 21. $\frac{1}{2}$ 22. 6
23. 2 24. -5 25. 0 26. $\frac{1}{2}$ 27. 2 28. 0 29. 1 30. 3 31. 4 32. -3 33. there is no solution
34. -5 35. all real numbers are solution 36. 0 37. 4 38. 9 units

Sample Problems



Solutions

1. $2x + 3 = 4x + 9$

Solution:

$$2x + 3 = 4x + 9 \quad \text{subtract } 2x \text{ from both sides}$$

$$3 = 2x + 9 \quad \text{subtract } 9 \text{ from both sides}$$

$$-6 = 2x \quad \text{divide both sides by } 2$$

$$-3 = x$$

We check: if $x = -3$, then

$$\text{LHS} = 2(-3) + 3 = -6 + 3 = -3$$

$$\text{RHS} = 4(-3) + 9 = -12 + 9 = -3$$

Thus our solution, $x = -3$ is correct. (Note: LHS is short for the left-hand side and RHS is short for the right-hand side.)

2. $3w - 5 = 5(w + 1)$

Solution: we first apply the law of distributivity to simplify the right-hand side.

$$3w - 5 = 5(w + 1)$$

$$3w - 5 = 5w + 5 \quad \text{subtract } 3w \text{ from both sides}$$

$$-5 = 2w + 5 \quad \text{subtract } 5 \text{ from both sides}$$

$$-10 = 2w \quad \text{divide both sides by } 2$$

$$-5 = w$$

We check. If $w = -5$, then

$$\text{LHS} = 3(-5) - 5 = -15 - 5 = -20$$

$$\text{RHS} = 5((-5) + 1) = 5(-4) = -20$$

Thus our solution, $w = -5$ is correct.

3. $3y - 9 = -2y + 4$

Solution:

$$3y - 9 = -2y + 4 \quad \text{add } 2y \text{ to both sides}$$

$$5y - 9 = 4 \quad \text{add } 9 \text{ to both sides}$$

$$5y = 13 \quad \text{divide both sides by } 5$$

$$y = \frac{13}{5}$$

We check. If $y = \frac{13}{5}$, then

$$\text{LHS} = 3\left(\frac{13}{5}\right) - 9 = \frac{3}{1} \cdot \frac{13}{5} - 9 = \frac{39}{5} - \frac{9}{1} = \frac{39}{5} - \frac{45}{5} = \frac{-6}{5} = -\frac{6}{5}$$

$$\text{RHS} = -2\left(\frac{13}{5}\right) + 4 = \frac{-2}{1} \cdot \frac{13}{5} + \frac{4}{1} = \frac{-26}{5} + \frac{20}{5} = \frac{-6}{5} = -\frac{6}{5}$$

Thus $y = \frac{13}{5}$ is the correct solution.

4. $4 - x = 3(x - 7)$

Solution: We first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 4 - x &= 3(x - 7) && \text{distribute } 3 \\ 4 - x &= 3x - 21 && \text{add } x \text{ to both sides} \\ 4 &= 4x - 21 && \text{add } 21 \text{ to both sides} \\ 25 &= 4x && \text{divide both sides by } 4 \\ \frac{25}{4} &= x \end{aligned}$$

We check. If $x = \frac{25}{4}$, then

$$\text{LHS} = 4 - x = 4 - \frac{25}{4} = \frac{4}{1} - \frac{25}{4} = \frac{16}{4} - \frac{25}{4} = \frac{16 - 25}{4} = \frac{-9}{4} = -\frac{9}{4}$$

$$\begin{aligned} \text{RHS} &= 3(x - 7) = 3\left(\frac{25}{4} - 7\right) = 3\left(\frac{25}{4} - \frac{7}{1}\right) = 3\left(\frac{25}{4} - \frac{28}{4}\right) = 3\left(\frac{25 - 28}{4}\right) \\ &= 3\left(\frac{-3}{4}\right) = \frac{3}{1} \cdot \frac{-3}{4} = \frac{-9}{4} = -\frac{9}{4} \end{aligned}$$

Thus our solution, $x = \frac{25}{4}$ is correct.

5. $7(j - 5) + 9 = 2(-2j + 5) + 5j$

Solution:

$$\begin{aligned} 7(j - 5) + 9 &= 2(-2j + 5) + 5j && \text{distribute on both sides} \\ 7j - 35 + 9 &= -4j + 10 + 5j && \text{combine like terms} \\ 7j - 26 &= j + 10 && \text{subtract } j \\ 6j - 26 &= 10 && \text{add } 26 \\ 6j &= 36 && \text{divide by } 6 \\ j &= 6 \end{aligned}$$

We check: if $j = 6$, then

$$\text{LHS} = 7(6 - 5) + 9 = 7 \cdot 1 + 9 = 7 + 9 = 16$$

$$\text{RHS} = 2(-2 \cdot 6 + 5) + 5 \cdot 6 = 2(-12 + 5) + 30 = 2(-7) + 30 = -14 + 30 = 16$$

Thus our solution, $j = 6$ is correct.

6. $3(x-5) - 5(x-1) = -2x + 1$

Solution:

$$3(x-5) - 5(x-1) = -2x + 1 \quad \text{multiply out parentheses}$$

$$3x - 15 - 5x + 5 = -2x + 1 \quad \text{combine like terms}$$

$$-2x - 10 = -2x + 1 \quad \text{add } 2x$$

$$-10 = 1$$

Since x disappeared from the equation and we are left with an unconditionally false statement, there is no solution for this equation. This type of an equation is called a **contradiction**.

7. $\frac{3-x}{4} - \frac{10-3x}{5} = x+2$

Solution:

$$\frac{3-x}{4} - \frac{10-3x}{5} = x+2 \quad \text{make everything a fraction}$$

$$\frac{3-x}{4} - \frac{10-3x}{5} = \frac{x+2}{1} \quad \text{common denominator}$$

$$\frac{5(3-x)}{20} - \frac{4(10-3x)}{20} = \frac{20(x+2)}{20} \quad \text{multiply by 20}$$

$$5(3-x) - 4(10-3x) = 20(x+2) \quad \text{distribute}$$

$$15 - 5x - 40 + 12x = 20x + 40 \quad \text{combine like terms}$$

$$7x - 25 = 20x + 40 \quad \text{subtract } 7x$$

$$-25 = 13x + 40 \quad \text{subtract } 40$$

$$-65 = 13x \quad \text{divide by 13}$$

$$-5 = x$$

We check:

$$\text{LHS} = \frac{3 - (-5)}{4} - \frac{10 - 3(-5)}{5} = \frac{8}{4} - \frac{25}{5} = 2 - 5 = -3$$

$$\text{RHS} = -5 + 2 = -3$$

Thus our solution, $x = -5$ is correct.

8. $\frac{3x+17}{2} = x-1 + \frac{x+19}{2}$

Solution:

$$\frac{3x+17}{2} = x-1 + \frac{x+19}{2} \quad \text{express everything as a fraction}$$

$$\frac{3x+17}{2} = \frac{x-1}{1} + \frac{x+19}{2} \quad \text{bring everything to the common denominator}$$

$$\frac{3x+17}{2} = \frac{2(x-1)}{2} + \frac{x+19}{2} \quad \text{add fractions on right hand side}$$

$$\begin{aligned}\frac{3x+17}{2} &= \frac{2(x-1)+x+19}{2} && \text{multiply out parentheses} \\ \frac{3x+17}{2} &= \frac{2x-2+x+19}{2} && \text{combine like terms} \\ \frac{3x+17}{2} &= \frac{3x+17}{2} && \text{multiply by 2} \\ 3x+17 &= 3x+17\end{aligned}$$

Because the left hand side is now identical to the right hand side, this equation is an identity, and all real numbers are solution.

9. $\frac{2}{3}(x-1) = \frac{3}{5}(x-4) + 1$

Solution: We re-write the expressions as fractions.

$$\begin{aligned}\frac{2(x-1)}{3} &= \frac{3(x-4)}{5} + \frac{1}{1} && \text{common denominator is 15} \\ \frac{5 \cdot 2(x-1)}{5 \cdot 3} &= \frac{3 \cdot 3(x-4)}{3 \cdot 5} + \frac{15}{15} \\ \frac{10(x-1)}{15} &= \frac{9(x-4)}{15} + \frac{15}{15} && \text{multiply by 15} \\ 10(x-1) &= 9(x-4) + 15 && \text{distribute} \\ 10x - 10 &= 9x - 36 + 15 && \text{combine like terms} \\ 10x - 10 &= 9x - 21 && \text{subtract } 9x \\ x - 10 &= -21 && \text{add 10} \\ x &= -11\end{aligned}$$

We check. If $x = -11$, then

$$\begin{aligned}\text{LHS} &= \frac{2}{3}(-11-1) = \frac{2}{3}(-12) = -8 \\ \text{RHS} &= \frac{3}{5}(-11-4) + 1 = \frac{3}{5}(-15) + 1 = -9 + 1 = -8\end{aligned}$$

Thus our solution, $x = -11$ is correct.

10. $\frac{2}{3}(x-7) = \frac{4}{5}(x+1)$

Solution:

$$\begin{aligned}\frac{2}{3}(x-7) &= \frac{4}{5}(x+1) \\ \frac{2}{3} \cdot \frac{x-7}{1} &= \frac{4}{5} \cdot \frac{x+1}{1} \\ \frac{2(x-7)}{3} &= \frac{4(x+1)}{5} && \text{bring fractions to common denominator} \\ \frac{5 \cdot 2(x-7)}{15} &= \frac{3 \cdot 4(x+1)}{15} && \text{multiply both sides by 15}\end{aligned}$$

$$\begin{array}{rcll}
 10(x-7) & = & 12(x+1) & \text{multiply out parentheses} \\
 10x-70 & = & 12x+12 & \text{subtract } 10x \\
 -70 & = & 2x+12 & \text{subtract } 12 \\
 -82 & = & 2x & \text{divide by } 2 \\
 -41 & = & x &
 \end{array}$$

We check:

$$\begin{array}{rcl}
 \text{LHS} & = & \frac{2}{3}(-41-7) = \frac{2}{3}(-48) = -32 \\
 \text{RHS} & = & \frac{4}{5}(-41+1) = \frac{4}{5}(-40) = -32
 \end{array}$$

Thus our solution, $x = -41$ is correct.

11. $\frac{x+2}{4} - \frac{x-3}{5} = 20-x$

Solution:

$$\begin{array}{rcll}
 \frac{x+2}{4} - \frac{x-3}{5} & = & 20-x & \text{make everything a fraction} \\
 \frac{x+2}{4} - \frac{x-3}{5} & = & \frac{20-x}{1} & \text{common denominator is } 20 \\
 \frac{5(x+2)}{20} - \frac{4(x-3)}{20} & = & \frac{20(20-x)}{20} & \text{multiply by } 20 \\
 5(x+2) - 4(x-3) & = & 20(20-x) & \text{distribute} \\
 5x+10-4x+12 & = & 400-20x & \text{combine like terms} \\
 x+22 & = & -20x+400 & \text{add } 20x \\
 21x+22 & = & 400 & \text{subtract } 22 \\
 21x & = & 378 & \text{divide by } 21 \\
 x & = & 18 &
 \end{array}$$

We check. If $x = 18$, then

$$\begin{array}{rcl}
 \text{LHS} & = & \frac{18+2}{4} - \frac{18-3}{5} = \frac{20}{4} - \frac{15}{5} = 5-3 = 2 \\
 \text{RHS} & = & 20-18 = 2
 \end{array}$$

Thus $x = 18$ is indeed the solution.

$$12. (x-3)^2 - (2x-5)(x+1) = 5 - (x-1)^2$$

Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must keep the parentheses.

$$\begin{aligned}
 (x-3)^2 - (2x-5)(x+1) &= 5 - (x-1)^2 \\
 x^2 - 3x - 3x + 9 - (2x^2 + 2x - 5x - 5) &= 5 - (x^2 - x - x + 1) && \text{combine like terms} \\
 x^2 - 6x + 9 - (2x^2 - 3x - 5) &= 5 - (x^2 - 2x + 1) && \text{distribute} \\
 x^2 - 6x + 9 - 2x^2 + 3x + 5 &= 5 - x^2 + 2x - 1 && \text{combine like terms} \\
 -x^2 - 3x + 14 &= -x^2 + 2x + 4 && \text{add } x^2 \\
 -3x + 14 &= 2x + 4 && \text{add } 3x \\
 14 &= 5x + 4 && \text{subtract } 4 \\
 10 &= 5x && \text{divide by } 5 \\
 2 &= x
 \end{aligned}$$

We check. If $x = 2$, then

$$\begin{aligned}
 \text{LHS} &= (2-3)^2 - (2 \cdot 2 - 5)(2+1) = (-1)^2 - (4-5)(2+1) = (-1)^2 - (-1) \cdot 3 \\
 &= 1 - (-3) = 4 \\
 \text{RHS} &= 5 - (2-1)^2 = 5 - 1^2 = 5 - 1 = 4
 \end{aligned}$$

Thus $\boxed{x=2}$ is indeed the solution.

$$13. (x+1)^2 - (2x-1)^2 + (3x)^2 = 6x(x-2)$$

Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must keep the parentheses.

$$\begin{aligned}
 (x+1)^2 - (2x-1)^2 + (3x)^2 &= 6x(x-2) \\
 x^2 + x + x + 1 - (4x^2 - 2x - 2x + 1) + 9x^2 &= 6x^2 - 12x \\
 x^2 + 2x + 1 - (4x^2 - 4x + 1) + 9x^2 &= 6x^2 - 12x && \text{distribute} \\
 x^2 + 2x + 1 - 4x^2 + 4x - 1 + 9x^2 &= 6x^2 - 12x && \text{combine like terms} \\
 6x^2 + 6x &= 6x^2 - 12x && \text{subtract } 6x^2 \\
 6x &= -12x && \text{add } 12x \\
 18x &= 0 && \text{divide by } 18 \\
 x &= 0
 \end{aligned}$$

We check. If $x = 0$, then

$$\begin{aligned}
 \text{LHS} &= (0+1)^2 - (2 \cdot 0 - 1)^2 + (3 \cdot 0)^2 = 1^2 - (-1)^2 + (0)^2 \\
 &= 1 - 1 + 0 = 0 \\
 \text{RHS} &= 6 \cdot 0 \cdot (0-2) = 6 \cdot 0 \cdot (-2) = 0
 \end{aligned}$$

Thus $\boxed{x=0}$ is indeed the solution.

$$14. 12 - (2p-1)(p+1) = -2(-p+5)^2$$

Solution: We first multiply the polynomials as indicated. If the product is subtracted or further multiplied, we must

keep the parentheses.

$$\begin{aligned}
 12 - (2p - 1)(p + 1) &= -2(-p + 5)^2 \\
 12 - (2p^2 + 2p - p - 1) &= -2(p^2 - 5p + 25) && \text{combine like terms} \\
 12 - (2p^2 + p - 1) &= -2(p^2 - 10p + 25) && \text{distribute} \\
 12 - 2p^2 - p + 1 &= -2p^2 + 20p - 50 && \text{combine like terms} \\
 -2p^2 - p + 13 &= -2p^2 + 20p - 50 && \text{add } 2p^2 \\
 -p + 13 &= 20p - 50 && \text{add } p \\
 13 &= 21p - 50 && \text{add } 50 \\
 63 &= 21p && \text{divide by } 21 \\
 3 &= p
 \end{aligned}$$

We check. If $p = 3$, then

$$\begin{aligned}
 \text{LHS} &= 12 - (2 \cdot 3 - 1)(3 + 1) = 12 - (6 - 1)(3 + 1) = 12 - 5 \cdot 4 = 12 - 20 = -8 \\
 \text{RHS} &= -2(-3 + 5)^2 = -2 \cdot 2^2 = -2 \cdot 4 = -8
 \end{aligned}$$

Thus $p = 3$ is indeed the solution.

15. If we increase all sides of a square by 3 units, the area of the square increase by 75 units. How long are the sides of the square?

Solution: Let us denote the side of the square by x . Then the increased side is $x + 3$. The equation will express the difference between the areas.

$$\begin{aligned}
 (x + 3)^2 &= x^2 + 75 && \text{expand complete square} \\
 x^2 + 6x + 9 &= x^2 + 75 && \text{subtract } x^2 \\
 6x + 9 &= 75
 \end{aligned}$$

This equation started out as a degree two equation but the quadratic terms cancelled out, and we are left with a linear equation.

$$\begin{aligned}
 6x + 9 &= 75 && \text{subtract } 9 \\
 6x &= 66 && \text{divide by } 6 \\
 x &= 11
 \end{aligned}$$

We check: If a square has sides 11 units long, its area is 121 unit^2 . After the increase, the sides are 14 units long, and so the area is 196 unit^2 . The difference between 196 and 121 is indeed 75, so our solution is correct. The sides of the square were 11 units long.

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