

Sample Problems

Solve each of the following equations. Make sure to check your solutions.

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|-------------------------|--|---|
| 1.) $2x + 3 = 4x + 9$ | 5.) $7(j - 5) + 9 = 2(-2j + 5) + 5j$ | 9.) $\frac{2}{3}(x - 1) = \frac{3}{5}(x - 4) + 1$ |
| 2.) $3w - 5 = 5(w + 1)$ | 6.) $3(x - 5) - 5(x - 1) = -2x + 1$ | 10.) $\frac{2}{3}(x - 7) = \frac{4}{5}(x + 1)$ |
| 3.) $3y - 9 = -2y + 4$ | 7.) $\frac{3 - x}{4} - \frac{10 - 3x}{5} = x + 2$ | 11.) $\frac{x + 2}{4} - \frac{x - 3}{5} = 20 - x$ |
| 4.) $4 - x = 3(x - 7)$ | 8.) $\frac{3x + 17}{2} = x - 1 + \frac{x + 19}{2}$ | |

Practice Problems

Solve each of the following equations. Make sure to check your solutions.

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|---|--|
| 1.) $5x - 3 = x + 9$ | 14.) $3(2x - 1) - 5(2 - x) = 4(x - 1) + 5$ |
| 2.) $-x + 13 = 2x + 1$ | 15.) $5(x - 1) - 3(x + 1) = 3x - 8$ |
| 3.) $-2x + 4 = 5x - 10$ | 16.) $5(x - 1) - 3(-x + 1) = -3 + 8x$ |
| 4.) $5x - 7 = 6x + 8$ | 17.) $-2x - (3x - 1) = 2(5 - 3x)$ |
| 5.) $8x - 1 = 3x + 19$ | 18.) $3(x - 4) - 4(x - 3) = 3(x - 2) + 2(3 - x)$ |
| 6.) $-7x - 1 = 3x - 21$ | 19.) $\frac{3x - 1}{5} - \frac{7 - x}{3} = 2x + 6$ |
| 7.) $3(x - 4) + 5(x + 8) = 2(x - 1)$ | 20.) $8(x - 3) - 3(5 - 2x) = x$ |
| 8.) $3(x - 4) = 2(x + 5)$ | 21.) $\frac{3x - 1}{4} + \frac{8 - 4x}{3} = -3 - x$ |
| 9.) $4(5x + 1) = 6x + 4$ | 22.) $\frac{3x - 2}{5} + \frac{x + 4}{3} = \frac{14(x + 1)}{15}$ |
| 10.) $3(2x - 7) - 2(5x + 2) = -5x - 30$ | 23.) $\frac{3}{8}x + 1\frac{4}{5} = \frac{1}{4}x + 1\frac{3}{10}$ |
| 11.) $a - 3 = 5(a - 1) - 2$ | 24.) $\frac{2x + 1}{3} - \frac{3 - x}{2} = x - 2$ |
| 12.) $3y - 2 = -2y + 18$ | 25.) $\frac{2}{3}x - 1 = -\frac{2}{3}\left(x + \frac{1}{2}\right)$ |
| 13.) $2(b + 1) - 5(b - 3) = 2(b - 7) + 1$ | |

Sample Problems - Answers

- 1.) -3 2.) -5 3.) $\frac{13}{5}$ 4.) $\frac{25}{4}$ 5.) 6 6.) no solution 7.) -5
8.) identity, all numbers are solution 9.) -11 10.) -41 11.) 18

Practice Problems - Answers

- 1.) 3 2.) 4 3.) 2 4.) -15 5.) 4 6.) 2 7.) -5 8.) 22 9.) 0
10.) -5 11.) 1 12.) 4 13.) 6 14.) 2 15.) 0 16.) contradiction, there is no solution
17.) 9 18.) 0 19.) -8 20.) 3 21.) -13 22.) identity, all numbers are solution
23.) -4 24.) -5 25.) $\frac{1}{2}$

Sample Problems - Solutions

1. $2x + 3 = 4x + 9$

Solution:

$$\begin{array}{rcll}
 2x + 3 & = & 4x + 9 & \text{subtract } 2x \text{ from both sides} \\
 3 & = & 2x + 9 & \text{subtract } 9 \text{ from both sides} \\
 -6 & = & 2x & \text{divide both sides by 2} \\
 -3 & = & x &
 \end{array}$$

We check: if $x = -3$, then

$$\begin{array}{rcl}
 \text{RHS} & = & 2(-3) + 3 = -6 + 3 = -3 \\
 \text{LHS} & = & 4(-3) + 9 = -12 + 9 = -3
 \end{array}$$

Thus our solution, $x = -3$ is correct.

2. $3w - 5 = 5(w + 1)$

Solution: we first apply the law of distributivity to simplify the right-hand side.

$$\begin{array}{rcll}
 3w - 5 & = & 5(w + 1) & \\
 3w - 5 & = & 5w + 5 & \text{subtract } 3w \text{ from both sides} \\
 -5 & = & 2w + 5 & \text{subtract } 5 \text{ from both sides} \\
 -10 & = & 2w & \text{divide both sides by 2} \\
 -5 & = & w &
 \end{array}$$

We check. If $w = -5$, then

$$\begin{array}{rcl}
 \text{LHS} & = & 3(-5) - 5 = -15 - 5 = -20 \\
 \text{RHS} & = & 5((-5) + 1) = 5(-4) = -20
 \end{array}$$

Thus our solution, $w = -5$ is correct.

3. $3y - 9 = -2y + 4$

Solution:

$$\begin{array}{rcll}
 3y - 9 & = & -2y + 4 & \text{add } 2y \text{ to both sides} \\
 5y - 9 & = & 4 & \text{add } 9 \text{ to both sides} \\
 5y & = & 13 & \text{divide both sides by } 5 \\
 y & = & \frac{13}{5} &
 \end{array}$$

We check. If $x = \frac{13}{5}$, then

$$\begin{array}{rcl}
 \text{LHS} & = & 3\left(\frac{13}{5}\right) - 9 = \frac{3}{1} \cdot \frac{13}{5} - 9 = \frac{39}{5} - \frac{9}{1} = \frac{39}{5} - \frac{45}{5} = \frac{-6}{5} = -\frac{6}{5} \\
 \text{RHS} & = & -2\left(\frac{13}{5}\right) + 4 = \frac{-2}{1} \cdot \frac{13}{5} + \frac{4}{1} = \frac{-26}{5} + \frac{20}{5} = \frac{-6}{5} = -\frac{6}{5}
 \end{array}$$

Thus $x = \frac{13}{5}$ is the correct solution.

4. $4 - x = 3(x - 7)$

Solution: We first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned}
 4 - x &= 3(x - 7) && \text{distribute } 3 \\
 4 - x &= 3x - 21 && \text{add } x \text{ to both sides} \\
 4 &= 4x - 21 && \text{add } 21 \text{ to both sides} \\
 25 &= 4x && \text{divide both sides by } 4 \\
 \frac{25}{4} &= x
 \end{aligned}$$

We check. If $x = \frac{25}{4}$, then

$$\begin{aligned}
 \text{LHS} &= 4 - x = 4 - \frac{25}{4} = \frac{4}{1} - \frac{25}{4} = \frac{16}{4} - \frac{25}{4} = \frac{16 - 25}{4} = \frac{-9}{4} = -\frac{9}{4} \\
 \text{RHS} &= 3(x - 7) = 3\left(\frac{25}{4} - 7\right) = 3\left(\frac{25}{4} - \frac{7}{1}\right) = 3\left(\frac{25}{4} - \frac{28}{4}\right) = 3\left(\frac{25 - 28}{4}\right) \\
 &= 3\left(\frac{-3}{4}\right) = \frac{3}{1} \cdot \frac{-3}{4} = \frac{-9}{4} = -\frac{9}{4}
 \end{aligned}$$

Thus our solution, $x = \frac{25}{4}$ is correct.

5. $7(j - 5) + 9 = 2(-2j + 5) + 5j$

Solution:

$$\begin{aligned}
 7(j - 5) + 9 &= 2(-2j + 5) + 5j && \text{distribute on both sides} \\
 7j - 35 + 9 &= -4j + 10 + 5j && \text{combine like terms} \\
 7j - 26 &= j + 10 && \text{subtract } j \\
 6j - 26 &= 10 && \text{add } 26 \\
 6j &= 36 && \text{divide by } 6 \\
 j &= 6
 \end{aligned}$$

We check: if $j = 6$, then

$$\begin{aligned}
 \text{LHS} &= 7(6 - 5) + 9 = 7 \cdot 1 + 9 = 7 + 9 = 16 \\
 \text{RHS} &= 2(-2 \cdot 6 + 5) + 5 \cdot 6 = 2(-12 + 5) + 30 = 2(-7) + 30 = -14 + 30 = 16
 \end{aligned}$$

Thus our solution is correct.

6. $3(x - 5) - 5(x - 1) = -2x + 1$

Solution:

$$\begin{aligned}
 3(x - 5) - 5(x - 1) &= -2x + 1 && \text{multiply out parentheses} \\
 3x - 15 - 5x + 5 &= -2x + 1 && \text{combine like terms} \\
 -2x - 10 &= -2x + 1 && \text{add } 2x \\
 -10 &= 1
 \end{aligned}$$

Since x disappeared from the equation and we are left with an unconditionally false statement, there is no solution for this equation. This type of an equation is called a **contradiction**.

7. $\frac{3-x}{4} - \frac{10-3x}{5} = x+2$
Solution:

$$\begin{aligned} \frac{3-x}{4} - \frac{10-3x}{5} &= x+2 && \text{make everything a fraction} \\ \frac{3-x}{4} - \frac{10-3x}{5} &= \frac{x+2}{1} && \text{common denominator} \\ \frac{5(3-x)}{20} - \frac{4(10-3x)}{20} &= \frac{20(x+2)}{20} && \text{multiply by 20} \\ 5(3-x) - 4(10-3x) &= 20(x+2) && \text{distribute} \\ 15 - 5x - 40 + 12x &= 20x + 40 && \text{combine like terms} \\ 7x - 25 &= 20x + 40 && \text{subtract } 7x \\ -25 &= 13x + 40 && \text{subtract } 40 \\ -65 &= 13x && \text{divide by 13} \\ -5 &= x \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= \frac{3 - (-5)}{4} - \frac{10 - 3(-5)}{5} = \frac{8}{4} - \frac{25}{5} = 2 - 5 = -3 \\ \text{RHS} &= -5 + 2 = -3 \end{aligned}$$

Thus our solution, -5 is correct.

8. $\frac{3x+17}{2} = x-1 + \frac{x+19}{2}$
Solution:

$$\begin{aligned} \frac{3x+17}{2} &= x-1 + \frac{x+19}{2} && \text{express everything as a fraction} \\ \frac{3x+17}{2} &= \frac{x-1}{1} + \frac{x+19}{2} && \text{bring everything to the common denominator} \\ \frac{3x+17}{2} &= \frac{2(x-1)}{2} + \frac{x+19}{2} && \text{add fractions on right hand side} \end{aligned}$$

$$\begin{aligned} \frac{3x+17}{2} &= \frac{2(x-1) + x+19}{2} && \text{multiply out parentheses} \\ \frac{3x+17}{2} &= \frac{2x-2+x+19}{2} && \text{combine like terms} \\ \frac{3x+17}{2} &= \frac{3x+17}{2} && \text{multiply by 2} \\ 3x+17 &= 3x+17 \end{aligned}$$

Because the left hand side is now identical to the right hand side, this equation is an identity, and all real numbers are solution.

$$9. \frac{2}{3}(x-1) = \frac{3}{5}(x-4) + 1$$

Solution: We re-write the expressions as fractions.

$$\begin{aligned} \frac{2(x-1)}{3} &= \frac{3(x-4)}{5} + \frac{1}{1} && \text{common denominator is 15} \\ \frac{5 \cdot 2(x-1)}{5 \cdot 3} &= \frac{3 \cdot 3(x-4)}{3 \cdot 5} + \frac{15}{15} \\ \frac{10(x-1)}{15} &= \frac{9(x-4)}{15} + \frac{15}{15} && \text{multiply by 15} \\ 10(x-1) &= 9(x-4) + 15 && \text{distribute} \\ 10x - 10 &= 9x - 36 + 15 && \text{combine like terms} \\ 10x - 10 &= 9x - 21 && \text{subtract } 9x \\ x - 10 &= -21 && \text{add 21} \\ x &= -11 \end{aligned}$$

We check. If $x = -11$, then

$$\begin{aligned} \text{LHS} &= \frac{2}{3}(-11-1) = \frac{2}{3}(-12) = -8 \\ \text{RHS} &= \frac{3}{5}(-11-4) + 1 = \frac{3}{5}(-15) + 1 = -9 + 1 = -8 \end{aligned}$$

Thus our solution, -11 is correct.

$$10. \frac{2}{3}(x-7) = \frac{4}{5}(x+1)$$

Solution:

$$\begin{aligned} \frac{2}{3}(x-7) &= \frac{4}{5}(x+1) \\ \frac{2}{3} \cdot \frac{x-7}{1} &= \frac{4}{5} \cdot \frac{x+1}{1} \\ \frac{2(x-7)}{3} &= \frac{4(x+1)}{5} && \text{bring fractions to common denominator} \\ \frac{5 \cdot 2(x-7)}{15} &= \frac{3 \cdot 4(x+1)}{15} && \text{multiply both sides by 15} \end{aligned}$$

$$\begin{aligned} 10(x-7) &= 12(x+1) && \text{multiply out parentheses} \\ 10x - 70 &= 12x + 12 && \text{subtract } 10x \\ -70 &= 2x + 12 && \text{subtract 12} \\ -82 &= 2x && \text{divide by 2} \\ -41 &= x \end{aligned}$$

We check:

$$\begin{aligned} \text{LHS} &= \frac{2}{3}(-41-7) = \frac{2}{3}(-48) = -32 \\ \text{RHS} &= \frac{4}{5}(-41+1) = \frac{4}{5}(-40) = -32 \end{aligned}$$

Thus our solution, -41 is correct.

11. $\frac{x+2}{4} - \frac{x-3}{5} = 20 - x$
 Solution:

$$\begin{array}{rcl}
 \frac{x+2}{4} - \frac{x-3}{5} & = & 20 - x & \text{make everything a fraction} \\
 \frac{x+2}{4} - \frac{x-3}{5} & = & \frac{20-x}{1} & \text{common denominator is 20} \\
 \frac{5(x+2)}{20} - \frac{4(x-3)}{20} & = & \frac{20(20-x)}{20} & \text{multiply by 20} \\
 5(x+2) - 4(x-3) & = & 20(20-x) & \text{distribute} \\
 5x + 10 - 4x + 12 & = & 400 - 20x & \text{combine like terms} \\
 x + 22 & = & -20x + 400 & \text{add } 20x \\
 21x + 22 & = & 400 & \text{subtract 22} \\
 21x & = & 378 & \text{divide by 21} \\
 x & = & 18 &
 \end{array}$$

We check. If $x = 18$, then

$$\begin{array}{rcl}
 \text{LHS} & = & \frac{18+2}{4} - \frac{18-3}{5} = \frac{20}{4} - \frac{15}{5} = 5 - 3 = 2 \\
 \text{RHS} & = & 20 - 18 = 2
 \end{array}$$

Thus 18 is indeed the solution.