

We will now continue studying algebraic expressions. In this section, we will learn how to simplify complicated algebraic expressions. We already know how to evaluate algebraic expressions. However, that is not always desirable or even possible. One frequently occurring example for this is when we cannot evaluate an algebraic expression because we do not know the value of the unknown. In such cases, we often need to **simplify** algebraic expressions. Consider, for example, the algebraic expression $2x + 3x$. We can simplify $2x + 3x$ and just write $5x$ instead. This is also called **combining like terms**.

Example 1. Simplify each of the following by combining like terms.

$$\text{a) } 3x - 2 - 10x + x + 7 \quad \text{b) } 3m - 2n - 4 - 8m + n + 1 \quad \text{c) } ab - a + b \quad \text{d) } p + q + p - q$$

Solution: a) $3x$, $-10x$, and x are like terms, and -2 and 7 are like terms.

To combine like terms, we add the numbers (sign included!) that are multiplying the variable(s). Such a number is called the **coefficient**. When combining like terms, we add the coefficients.

To combine $3x$, $-10x$, and x , we add the coefficients: $3x - 10x + x = (3 - 10 + 1)x = -6x$

Caution! It is a common mistake to misinterpret $3 - 10 + 1$ as $3 - 11$. Not so!

To combine -2 and 7 , we just add and so we get 5 . The entire process can be done mentally, so our computation will look like this:

$$3x - 2 - 10x + x + 7 = \boxed{-6x + 5}$$

$$\text{b) } 3m - 2n - 4 - 8m + n + 1 = 3m - 8m - 2n + n - 4 + 1 = \boxed{-5m - n - 3}$$

-3 , $-n$, and $-5m$ are unlike terms and so this expression can not be further simplified.

$$\text{c) } \boxed{ab - a + b} \text{ since all three terms are unlike, so this expression can not be simplified.}$$

$$\text{d) } p + q + p - q = p + p + q - q = \boxed{2p}$$

In the last example, as we simplified the expression, one of the variables disappeared. This special and often celebrated case of combining like terms is what we call **cancellation**.

To add two or more algebraic expressions, we drop parentheses and combine like terms.

Example 2. Add the algebraic expressions as indicated.

$$\text{a) } (3a - 5b) + (2a - b) \quad \text{b) } (-m + 3n - 4) + (5m - n - 4) \quad \text{c) } (3y + 5) + (3y - 5)$$

$$\text{Solution: a) } (3a - 5b) + (2a - b) = 3a - 5b + 2a - b = 3a + 2a - 5b - b = \boxed{5a - 6b}$$

$$\text{b) } (-m + 3n - 4) + (5m - n - 4) = -m + 5m + 3n - n - 4 - 4 = \boxed{4m + 2n - 8}$$

$$\text{c) } (3y + 5) + (3y - 5) = 3y + 5 + 3y - 5 = 3y + 3y + 5 - 5 = \boxed{6y}$$

To multiply an algebraic expression by a number or a one-term expression, we apply the distributive law:

$$a(b + c) = ab + ac \quad \text{for all numbers } a, b, \text{ and } c.$$

Example 3. Expand the products as indicated.

$$\text{a) } 3(5a - b + 1) \quad \text{b) } -1(-x^2 + 3x - 4) \quad \text{c) } 5x(2a - x) \quad \text{d) } -ab(3a - 5b - 1) \quad \text{e) } -4(3m - 5)$$

Solution: a) $3(5a - b + 1) = 15a - 3b + 3$

b) $-1(-x + 3y - 4) = x - 3y + 4$

Note that multiplication by -1 means we change the sign in front of each term.

c) $5x(2a - 3) = 5x \cdot 2a - 5x \cdot 3 = 10ax - 15x = 10ax - 15x$

d) $-ab(3a - 5b - 1) = -ab \cdot 3a - ab(-5b) - ab(-1) = -3a^2b + 5ab^2 + ab$

e) $-4(3m - 5) = -12m + 20$

To subtract an algebraic expression, we add the opposite.

Example 4. Perform the subtractions between algebraic expressions as indicated.

$$\text{a) } (3a - 5b) - (2a - b) \quad \text{b) } (3y + 5) - (3y - 5)$$

Solution: a) We apply one fundamental fact of algebra: *To subtract is to add the opposite.* The opposite is always obtained by multiplication by -1 . And this multiplication by -1 means we need to distribute -1 . We subtract the entire expression, not just its first term. Here is the argument, broken down to logical steps.

$$\begin{aligned} (3a - 5b) - (2a - b) &= && \text{to subtract is to add the opposite} \\ &= (3a - 5b) + (-1)(2a - b) && \text{the opposite is obtained by multiplying by } -1 \\ &&& \text{(careful with the distributive law)} \\ &= (3a - 5b) + (-2a + b) && \text{we add the algebraic expressions by dropping the parentheses} \\ &= 3a - 5b - 2a + b && \text{and combine like terms} \\ &= 3a - 2a - 5b + b = a - 4b \end{aligned}$$

But this is way too much writing. While the idea is the same, our computation usually looks like this:

$$(3a - 5b) - (2a - b) = 3a - 5b - 2a + b = a - 4b$$

Careful! When computing on paper, we advise *not to subtract mentally*. To subtract is to add the opposite. Take the time of writing down the opposite, and then *add* mentally.

b) $(3y + 5) - (3y - 5) = (3y + 5) + (-3y + 5) = 10$

We can now combine more complicated expressions.

Example 5. Simplify each of the following expressions.

$$\text{a) } 4(a - 2b + 1) - 5(2a - b - 1) \quad \text{b) } 6(2y + 1) - 5(3y - 5) \quad \text{c) } 3(5x - 2) - 5(3x + 1)$$

Solution: a) We apply the distributive law and then combine like terms.

$$4(a - 2b + 1) - 5(2a - b - 1) = 4a - 8b + 4 - 10a + 5b + 5 = -6a - 3b + 9$$

b) $6(2y + 1) - 5(3y - 5) = 12y + 6 - 15y + 25 = -3y + 31$

c) $3(5x - 2) - 5(3x + 1) = 15x - 6 - 15x - 5 = -11$

The last example illustrates the benefits of algebra. If we were asked to evaluate the expression

$3(5x - 2) - 5(3x + 1)$ when $x = 8, -20$, or -97 , or any other number, we might be computing for minutes, but the result will always be -11 . In other words, this expression is **equivalent** to -11 . It is a natural instinct and *widely accepted convention to always present expressions in their simplest possible form.*



Discussion: Explain how $2x + 3x = 5x$ can be explained in terms of the distributive law.

Note: The equation $3a + 2a = 5a$ is an **identity**. An identity is an equation for which all numbers are solutions. We only check two values for a :

$$\text{If } a = 5, \text{ then } 3 \cdot 5 + 2 \cdot 5 = 15 + 10 = 25 = 5 \cdot 5$$

$$\text{If } a = -2, \text{ then } 3(-2) + 2(-2) = -6 + (-4) = -10 = 5(-2)$$

When we simplify algebraic expressions, equations such as $5x - 2x = 3x$ are always identities.

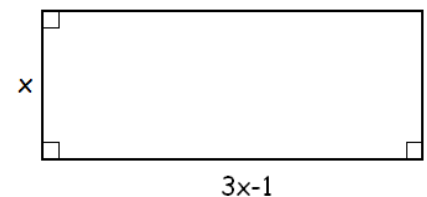
Example 6. The longer side of a rectangle is one foot shorter than three times the shorter side. Find the sides of the rectangle if we also know that its perimeter is 54 feet.

There is no denying that this problem is our first more complex real word problem (also called application problem). The general steps for solving such a problem are as follows.

1. Read the text of the problem. Make a decision about what quantity to label by x . Write this decision down.
2. Read the text of the problem. Label all other quantities in the problem in terms of x . This usually involves translations from English to algebraic expressions using x . Organize all this using a table or a picture.
3. Read the text of the problem. There should be one more piece of information that wasn't used yet. Based on that information, write an equation in x .
4. Solve the equation for x .
5. Now that we know the value of x , all other quantities in the problem can be found, because they were labeled in terms of x .
6. Read the text of the problem. Formulate your answer.
7. Check whether your solution satisfies all conditions stated in the problem.

Do not expect to solve the problem on the first attempt. We often have to try to call several things x before we can successfully solve the problem.

Solution: Let us denote the shorter side by x . Then the longer side can be expressed as $3x - 1$. Before we move on, it might be useful to organize our data using a table or a picture.



We will write and solve an equation expressing the perimeter of the rectangle. The perimeter of a rectangle with sides a and b is $P = 2a + 2b$.

$$2 \cdot \underbrace{x}_{\text{shorter side}} + 2 \cdot \underbrace{(3x - 1)}_{\text{longer side}} = 54$$

We will simplify the left-hand side by applying the distributive law and then combining like terms.

$$2x + 2(3x - 1) = 54 \quad \text{distribute 2}$$

$$2x + 6x - 2 = 54 \quad \text{combine like terms}$$

$$8x - 2 = 54$$

This is now a two-step equation we can easily solve for x .

$$8x - 2 = 54 \quad \text{add 2}$$

$$8x = 56 \quad \text{divide by 8}$$

$$x = 7$$

Now that we found x , we suddenly know everything, because x is 7 everywhere.

$$\text{shorter side: } x = 7$$

$$\text{longer side: } 3x - 1 = 3 \cdot 7 - 1 = 20$$

The shorter side was labeled x . We know now that x is 7. The longer side was labeled $3x - 1$. Since x is still 7, that means that the longer side is $3 \cdot 7 - 1 = 20$. Thus the rectangle has sides 7 feet and 20 feet long.

Good news! We do not need to check if 7 is indeed the solution of the equation. What if we correctly solved the wrong equation? Recall that we came up with the equation, it wasn't given. Instead, we should check if our solution satisfies the conditions stated in the problem. Is the longer side indeed one foot shorter than three times the shorter side? Indeed, $3 \cdot 7 - 1 = 20$ ✓ Is the perimeter 54 feet? Indeed,

$$2(7 \text{ ft}) + 2(20 \text{ ft}) = 14 \text{ ft} + 40 \text{ ft} = 54 \text{ ft}. \quad \checkmark \text{ So the sides are indeed } \boxed{7 \text{ feet and } 20 \text{ feet}} \text{ long.}$$

Example 7. When Candice took over the register, it contained 30 bills, all five-dollar and ten-dollar bills. How many of each were there if the value of all of these bills together was 210 dollars?

Solution: Suppose that we label the number of ten-dollar bills by x . Then we have $30 - x$ many five-dollar bills. (see Example 4.) We will set up the equation to express the total value of the bills.

	ten-dollar bills	five-dollar bills
number of bills	x	$30 - x$
value in dollars	$10x$	$5(30 - x)$

The equation will express the value of the bills. We solve the equation.

$$10x + 5(30 - x) = 210 \quad \text{distribute 5}$$

$$10x + 150 - 5x = 210 \quad \text{combine like terms}$$

$$5x + 150 = 210 \quad \text{subtract 150}$$

$$5x = 60 \quad \text{divide by 5}$$

$$x = 12$$

Since we labeled the number of ten-dollar bills by x , we have 12 ten-dollar bills. The number of five-dollar bills was labeled $30 - x$, thus there are $30 - 12 = 18$ five-dollar bills. Our answer is:

$\boxed{12 \text{ ten-dollar bills and } 18 \text{ five-dollar bills}}$. We check: $12 + 18 = 30$ so we indeed have 30 bills. The value of all bills is $12 \cdot 10 + 18 \cdot 5 = 120 + 90 = 210$ dollars. Thus our answer is correct.

Sometimes expressions are more complicated than the ones we just saw. One complication can be several pairs of parentheses nested inside each other. Just like in case of order of operations, we start with the innermost parentheses.

Example 8. Simplify the expression $-3(2(3x - 1) + 5 - 7x) + 10$

Solution: We will eliminate parentheses by applying the distributive law, combine like terms, and repeat. Note that if we take the time to combine like terms after expanding a multiplication, that will cut down on the computation in the next step.

$$\begin{aligned}
 -3(2(3x - 1) + 5 - 7x) + 10 &= && \text{distribute in innermost parentheses} \\
 &= -3((6x - 2) + 5 - 7x) + 10 && \text{drop parentheses, combine like terms} \\
 &= -3(-x + 3) + 10 && \text{apply the distributive law} \\
 &= 3x - 9 + 10 && \text{combine like terms} \\
 &= \boxed{3x + 1}
 \end{aligned}$$



Enrichment

Think of an integer. Add six to it. Double the sum. Take its opposite. Add eight. Multiply by five. Add ten times your original number. Add 25. Double again. No matter what number you had in mind, the final result is 10. Try this with a few numbers. Can you explain why this happens?



Sample Problems

1. Evaluate each of the following algebraic expressions with the value(s) given.
 - a) $-x^2 - 5x + 2$ if $x = -2$
 - b) $-16t^2 + 32t + 240$ if $t = 3$
2. Add the algebraic expressions as indicated.
 - a) $(3x - 5y) + (2x + 4y)$
 - b) $(2a - 5b + 3) + (-a - 8b + 3)$
3. Multiply the algebraic expressions by a number as indicated.
 - a) $3(2x + 4y - 5)$
 - b) $-5(2a - b + 8)$
 - c) $-1(-p + 3q - 8m + 6)$
 - d) $-(-a + 3b - 7)$
4. Subtract the algebraic expressions as indicated.
 - a) $(2a + b) - (a - b)$
 - b) $(3x - 5y) - (2x + 4y)$
 - c) $(2m - 5n + 3) - (-m - 8n + 3)$
 - d) $(2a - 2b) - (b - a)$
 - e) $(3a - 2) - (1 - 4a)$
5. Simplify each of the following.
 - a) $3(x - 5) - 5(x - 1)$
 - b) $4(2a - b) - 3(5a - 2b)$
 - c) $2(a - 2b) + 3(5b - 2a) - 4(2b - a)$
 - d) $-(3a - 2) - (1 - 4a)$
6. Simplify the given expression. $3(-2(-(6x - 1) + 3(2x - 5) - 5x + 7) - 8x - 10)$
7. Solve each of the given equations. Make sure to check your solutions.
 - a) $5(3x - 8) - 2(8x - 1) = -33$
 - b) $4(5x - 3(2x - 1) + 2x) = 52$
 - c) $2(3(4(5x - 1) - 17x + 2) - 3x + 1) = 14$
8. Solve each of the following application problems.
 - a) Ann and Betty dine together. The total bill is \$38. Ann paid \$2 more than Betty. How much did Betty pay?
 - b) 55 people showed up on the party. There were 3 less women than men. How many men were there?
 - c) One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.
 - d) One side of a rectangle is 3 cm longer than four times the other side. Find the sides if the perimeter of the rectangle is 216 cm.
 - e) The sum of two consecutive even integers is -170 . Find these numbers.
 - f) The sum of three consecutive odd integers is 57. Find these numbers.
 - g) Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?
 - h) We have a jar of coins, all quarters and dimes. All together, they are worth \$17.60 We have 13 more quarters than dimes. How many quarters, how many dimes?
 - i) Red pens cost \$2 each, blue ones cost \$3 each. We bought some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid \$116?



Practice Problems

1. Evaluate each of the following algebraic expressions with the value(s) given.

a) $3x^2 - x + 7$ if $x = -1$

b) $-a + 5b$ if $a = 3$ and $b = -2$

c) $\frac{x^x - 1}{x - 1}$ if $x = 2$

2. Add the algebraic expressions as indicated.

a) $(2a - 7y + 1) + (2a - 7y - 1)$

c) $(4p - 5q + 1) + (5p - 4q - 2)$

e) $(-5y + 8) + (5y - 8)$

b) $(-2a - 5) + (3a + 5)$

d) $(2x - 3y + 8) + (-2x - 3y - 8)$

3. Multiply the algebraic expressions by a number as indicated.

a) $5(-3a + b - 5)$

b) $0(2x - 7y + 8z)$

c) $-6(n - 3m - 8)$

d) $-(p - 5q + 3r - 1)$

4. Subtract the algebraic expressions as indicated.

a) $(2a - 7y) - (2a - 7y)$

c) $(2x - 5y - 1) - (-y + 1)$

b) $(-2a - 5b - 1) - (3a + 5b - 1)$

d) $(2m - 3n + 8) - (-2m - 3n - 8)$

5. Simplify each of the following.

a) $(x + 1) - (x - 1)$

c) $-2 + x - 3(x - 1) - (1 - 2x)$

b) $2(x - 1) - 3(x - 7)$

d) $-2(a - 3b + c) - 3(a - b + 4c)$

6. Simplify each of the given expressions.

a) $-2((3x - 8(-2x + 1) - 3x) - 5x + 2)$

b) $3(5(-2(-(2x - 1) + 1) - 2) + 3) - 7$

c) $2((5x - 1) - 3((8x + 1) + 4(x - 5) - x + 1) + 3x - 3)$

7. Solve each of the following equations. Make sure to check your solutions.

a) $3(x + 2) - 2(5x - 1) = 1 - 2(x + 4)$

b) $4(2(3(x - 5) + 7 - 2x) - x + 8) = -3(x - 8)$

c) $5(3(3x - 1) - 2(4x + 3)) = -7(2x + 9) - 1$

d) $-2(5(2x - 1) - 2(5x + 1) - 3x) + 3 = 2(x - 4) + 1$

e) $5x - 4(2x - 1) - 3(x - 2) = 5(2(3x - 1) - 4(x - 1))$

f) $7x - 2((3x + 5) - 3((4x + 1) - 5(x - 2)) - x) = -4$

8. Consider the rectangle shown on the picture. Express the perimeter and area of the rectangle in terms of x . Simplify the expressions.

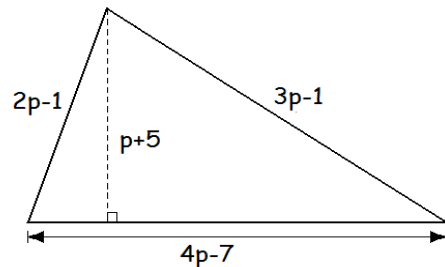
Units are not given. This means that the answer can be presented either without units, or perimeter in unit and area in unit². (For example, $P = 14$ unit and $A = 24$ unit²).



9. Solve each of the following application problems.

- a) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides of the rectangle if its perimeter is 64 ft.
- b) A bank teller has some five-dollar bills and ten-dollar bills. The number of five-dollar bills is ten less than twice the number of ten-dollar bills. The total value of the money is \$610. How much of each denomination of bill does he have?
- c) Sally worked 50 hours last week and made \$660 for the week. For every hour worked over 40 her job pays time and a half. What is Sally's regular hourly pay rate?
- d) One number is five greater than six times another number. Find these numbers if their sum is 33.
- e) Amy's age is three less than five times her son's age. How old are they if the sum of their ages is 33?
- f) Eight times a the sum of a number and six is 34 greater than the number. Find this number.

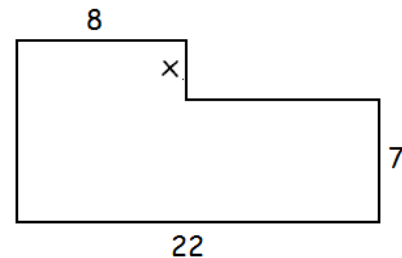
- g) Find the value of p if the triangle shown on the picture has a perimeter 54 units. Given the value of p you found, compute the area of the triangle.



- h) Three times a number is one less than twice the difference of the number and three. Find this number.

- i) There were a lot of coins in that jar, all quarters and dimes. The number of dimes was two less than five times the number of quarters. How many of each coins were there if all the coins in the jar were worth 8 dollars and 80 cents?
- j) Consider the figure shown on the picture. Angles that look like right angles are right angles. Find the value of x if we know that the area of this object is 178 unit^2 .

- k) The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost \$12, adult tickets cost \$19. The number of children ticket purchased was three more than four times the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of \$304 dollars?



- l) The sum of three consecutive multiples of 5 is 750. Find these numbers.
- m) Stan has five more textbooks than fiction books. How many of each does he have if all together he has 93 books?



Answers

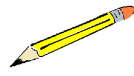
Sample Problems

1. a) 8 b) 192 2. a) $5x - y$ b) $a - 13b + 6$
3. a) $6x + 12y - 15$ b) $-10a + 5b - 40$ c) $p - 3q + 8m - 6$ d) $a - 3b + 7$
4. a) $a + 2b$ b) $x - 9y$ c) $3m + 3n$ d) $3a - 3b$ e) $7a - 3$
5. a) $-2x - 10$ b) $-7a + 2b$ c) $3b$ d) $a + 1$ 6. a) -5 b) 10 c) 2
7. $6x + 12$
8. a) Betty paid \$18 and Ann paid \$20. b) There were 26 women and 29 men on the party.
c) 11 cm by 48 cm d) 21 cm by 87 cm e) -86 and -84 f) 17, 19, and 21
g) 7 big and 17 small h) 41 dimes and 54 quarters i) 10 blue and 43 red pens

Practice Problems

1. a) 11 b) -13 c) 3 2. a) $4a - 14y$ b) a c) $9p - 9q - 1$ d) $-6y$ e) 0
3. a) $5b - 15a - 25$ b) 0 c) $-6n + 18m + 48$ d) $-p + 5q - 3r + 1$
4. a) 0 b) $-5a - 10b$ c) $2x - 4y - 2$ d) $4m + 16$
5. a) 2 b) $-x + 19$ c) 0 d) $-5a + 9b - 14c$ 6. a) $-22x + 12$ b) $60x - 88$ c) $-50x + 100$
7. a) 3 b) 8 c) -1 d) -6 e) 0 f) 20 8. $P = (6x + 22)$ unit $A = (36x - 12)$ unit²
9. a) 9 ft by 23 ft b) 33 ten-dollar bills and 56 five-dollar bills c) \$12 d) 4 and 29
e) 6 years and 27 years f) -2 g) $p = 7$, $A = 126$ unit² h) -7
i) 12 quarters and 58 dimes j) 3 k) 4 adult and 19 children l) 245, 250, 255
m) 44 fiction and 49 textbooks

Sample Problems



Solutions

1. Evaluate each of the following algebraic expressions with the value(s) given.

a) $-x^2 - 5x + 2$ if $x = -2$

Solution: We substitute -2 into the expression. Please note that if the value of x is negative, we will need to place parentheses around it.

$$-x^2 - 5x + 2 = -(-2)^2 - 5(-2) + 2$$

According to order of operations, we perform the exponentiation first.

$$-(-2)^2 - 5(-2) + 2 = -4 - 5(-2) + 2 = -4 - (-10) + 2 = -4 + 10 + 2 = 6 + 2 = \boxed{8}$$

Why don't the two minuses make a plus in $-(-2)^2$? They do, it's just that there are three minus signs and not two: $-(-2)^2 = -1(-2)(-2)$.

b) $-16t^2 + 32t + 240$ if $t = 3$

Solution: We substitute 3 into the expression. Please note that if the value of x is negative, we will need to place parentheses around it.

$$\begin{aligned} -16t^2 + 32t + 240 &= -16(3)^2 + 32(3) + 240 \\ &= -16 \cdot 9 + 32 \cdot 3 + 240 \\ &= -144 + 96 + 240 = -48 + 240 = \boxed{192} \end{aligned}$$

2. Add the algebraic expressions as indicated.

a) $(3x - 5y) + (2x + 4y)$

Solution: We drop the parentheses and combine like terms.

$$\begin{aligned} (3x - 5y) + (2x + 4y) &= \\ 3x - 5y + 2x + 4y &= && \text{organize like terms together} \\ 3x + 2x - 5y + 4y &= && 3 + 2 = 5 \quad \text{and} \quad -5 + 4 = -1 \\ &= \boxed{5x - y} \end{aligned}$$

b) $(2a - 5b + 3) + (-a - 8b + 3)$

Solution: We drop the parentheses and combine like terms.

$$\begin{aligned} (2a - 5b + 3) + (-a - 8b + 3) &= \\ 2a - 5b + 3 - a - 8b + 3 &= \\ 2a - a - 5b - 8b + 3 + 3 &= && 2 - 1 = 1, \quad -5 - 8 = -13, \quad \text{and} \quad 3 + 3 = 6 \\ 1a - 13b + 6 &= \boxed{a - 13b + 6} \end{aligned}$$

3. Multiply the algebraic expressions by a number as indicated.

a) $3(2x + 4y - 5)$

Solution: We distribute 3.

$$3(2x + 4y - 5) = \boxed{6x + 12y - 15}$$

b) $-5(2a - b + 8)$

Solution: We distribute -5 .

$$-5(2a - b + 8) = \boxed{-10a + 5b - 40}$$

c) $-1(-p + 3q - 8m + 6)$

Solution: We distribute -1 .

$$-1(-p + 3q - 8m + 6) = \boxed{p - 3q + 8m - 6}$$

d) $-(-a + 3b - 7)$

Solution: The notation here indicates multiplication by -1 , which is the same as taking the opposite of a quantity. We distribute -1 .

$$-1(-a + 3b - 7) = \boxed{a - 3b + 7}$$

4. Subtract the algebraic expressions as indicated.

a) $(2a + b) - (a - b)$

Solution: To subtract is to add the opposite. The opposite of $a - b$ is $-a + b$ since

$$-1(a - b) = -a + b$$

$$\begin{aligned} \text{Thus } (2a + b) - (a - b) &= (2a + b) + (-a + b) && \text{drop parentheses} \\ &= 2a + b - a + b && \text{combine like terms} \\ &= \boxed{a + 2b} \end{aligned}$$

b) $(3x - 5y) - (2x + 4y)$

Solution: To subtract is to add the opposite. The opposite of $2x + 4y$ is $-2x - 4y$ since

$$-1(2x + 4y) = -2x - 4y$$

$$\begin{aligned} (3x - 5y) - (2x + 4y) &= (3x - 5y) + (-2x - 4y) && \text{to subtract is to add the opposite} \\ &= 3x - 5y - 2x - 4y && \text{drop parentheses, combine like terms} \\ &= \boxed{x - 9y} \end{aligned}$$

c) $(2m - 5n + 3) - (-m - 8n + 3)$

Solution: To subtract is to add the opposite. The opposite of $-m - 8n + 3$ is $m + 8n - 3$ since

$$-1(-m - 8n + 3) = m + 8n - 3$$

$$\begin{aligned}(2m - 5n + 3) - (-m - 8n + 3) &= && \text{to subtract is to add the opposite} \\(2m - 5n + 3) + (m + 8n - 3) &= && \text{drop parentheses, combine like terms} \\2m - 5n + 3 + m + 8n - 3 &= && \boxed{3m + 3n}\end{aligned}$$

d) $(2a - 2b) - (b - a)$

Solution: To subtract is to add the opposite. The opposite of $b - a$ is $-b + a$ since

$$-1(b - a) = -b + a$$

$$\begin{aligned}(2a - 2b) - (b - a) &= (2a - 2b) + (-b + a) && \text{to subtract is to add the opposite} \\&= 2a - 2b - b + a && \text{drop parentheses, combine like terms} \\&= \boxed{3a - 3b}\end{aligned}$$

e) $(3a - 2) - (1 - 4a)$

Solution: To subtract is to add the opposite. The opposite of $1 - 4a$ is $-1 + 4a$ since

$$-1(1 - 4a) = -1 + 4a$$

$$\begin{aligned}(3a - 2) - (1 - 4a) &= && \text{to subtract is to add the opposite} \\(3a - 2) + (-1 + 4a) &= && \text{drop parentheses, combine like terms} \\3a - 2 - 1 + 4a &= && \boxed{7a - 3}\end{aligned}$$

5. Simplify each of the following.

a) $3(x - 5) - 5(x - 1)$

Solution: We apply the law of distributivity and combine like terms. Notice that the last term is $-5(-1) = 5$.

$$\begin{aligned}3(x - 5) - 5(x - 1) &= && \text{apply the distributive law} \\3x - 15 - 5x + 5 &= && \text{combine like terms} \\&= && \boxed{-2x - 10}\end{aligned}$$

b) $4(2a - b) - 3(5a - 2b)$

Solution: We apply the law of distributivity and combine like terms

$$\begin{aligned}4(2a - b) - 3(5a - 2b) &= 8a - 4b - 15a + 6b \\&= \boxed{-7a + 2b}\end{aligned}$$

c) $2(a - 2b) + 3(5b - 2a) - 4(2b - a)$

Solution: We apply the law of distributivity and combine like terms

$$2(a - 2b) + 3(5b - 2a) - 4(2b - a) = 2a - 4b + 15b - 6a - 8b + 4a = \boxed{3b}$$

d) $-(3a - 2) - (1 - 4a)$

Solution:

$$\begin{aligned} -(3a - 2) - (1 - 4a) &= -1(3a - 2) - 1(1 - 4a) && \text{multiplication} \\ &= -3a + 2 - 1 + 4a && \text{combine like terms} \\ &= \boxed{a + 1} \end{aligned}$$

6. Simplify the given expression. $3(-2(-(6x - 1) + 3(2x - 5) - 5x + 7) - 8x - 10)$

Solution: We first need to investigate the parentheses and pair them off. There is no point starting the computation without understanding them; it will only get worse later. We will denote the different parentheses with differently shaped grouping symbols. There are two innermost parentheses, one after the other. We start there, left to right.

$$\begin{aligned} 3\{-2[-(6x - 1) + 3(2x - 5) - 5x + 7] - 8x - 10\} &= && \text{distribute } -1 \\ &= 3\{-2[-6x + 1 + 3(2x - 5) - 5x + 7] - 8x - 10\} && \text{combine like terms} \\ &= 3\{-2[-11x + 8 + 3(2x - 5)] - 8x - 10\} && \text{distribute } 3 \\ &= 3\{-2[-11x + 8 + 6x - 15] - 8x - 10\} && \text{combine like terms} \\ &= 3\{-2(-5x - 7) - 8x - 10\} && \text{distribute } -2 \\ &= 3\{10x + 14 - 8x - 10\} && \text{combine like terms} \\ &= 3(2x + 4) && \text{distribute } 3 \\ &= \boxed{6x + 12} \end{aligned}$$

7. Solve each of the given equations. Make sure to check your solutions.

a) $5(3x - 8) - 2(8x - 1) = -33$

Solution: We need to simplify the left-hand side. After that, the equation will be a simple two-step equation.

$$\begin{aligned} 5(3x - 8) - 2(8x - 1) &= -33 && \text{distribute } 5 \text{ and } -2 \\ 15x - 40 - 16x + 2 &= -33 && \text{combine like terms} \\ -x - 38 &= -33 && \text{add } 38 \\ -x &= 5 && \text{divide (or multiply) by } -1 \\ x &= -5 \end{aligned}$$

We check: if $x = -5$, then

$$\begin{aligned} \text{LHS} &= 5[3(-5) - 8] - 2[8(-5) - 1] = 5(-15 - 8) - 2(-40 - 1) = 5(-23) - 2(-41) \\ &= -115 + 82 = -33 = \text{RHS } \checkmark \end{aligned}$$

Thus our solution, $\boxed{x = -5}$, is correct.

$$\text{b) } 4(5x - 3(2x - 1) + 2x) = 52$$

Solution: We need to simplify the left-hand side. After that, the equation will be a simple two-step equation. We start with the innermost parentheses.

$$\begin{array}{rcll} 4(5x - 3(2x - 1) + 2x) & = & 52 & \text{distribute } -3 \\ 4(5x - 6x + 3 + 2x) & = & 52 & \text{combine like terms} \\ 4(x + 3) & = & 52 & \text{distribute } 4 \\ 4x + 12 & = & 52 & \text{subtract } 12 \\ 4x & = & 40 & \text{divide by } 4 \\ x & = & 10 & \end{array}$$

We check: if $x = 10$, then

$$\begin{aligned} \text{LHS} &= 4(5 \cdot 10 - 3(2 \cdot 10 - 1) + 2 \cdot 10) = 4(5 \cdot 10 - 3(20 - 1) + 2 \cdot 10) = 4(5 \cdot 10 - 3 \cdot 19 + 2 \cdot 10) \\ &= 4(50 - 57 + 20) = 4(-7 + 20) = 4 \cdot 13 = 52 = \text{RHS } \checkmark \end{aligned}$$

$$\text{c) } 2(3(4(5x - 1) - 17x + 2) - 3x + 1) = 14$$

Solution: After we simplified the expression on the left-hand side, we will again have an easy two-step equation. The three pairs of parentheses are nested inside each other. We start with the innermost one.

$$\begin{array}{rcll} 2\{3[4(5x - 1) - 17x + 2] - 3x + 1\} & = & 14 & \text{distribute } 4 \\ 2\{3(20x - 4 - 17x + 2) - 3x + 1\} & = & 14 & \text{combine like terms} \\ 2\{3(3x - 2) - 3x + 1\} & = & 14 & \text{distribute } 3 \\ 2(9x - 6 - 3x + 1) & = & 14 & \text{combine like terms} \\ 2(6x - 5) & = & 14 & \text{distribute } 2 \\ 12x - 10 & = & 14 & \text{add } 10 \\ 12x & = & 24 & \text{divide by } 12 \\ x & = & 2 & \end{array}$$

We check: if $x = 2$, then

$$\begin{aligned} \text{LHS} &= 2\{3[4(5 \cdot 2 - 1) - 17 \cdot 2 + 2] - 3 \cdot 2 + 1\} = 2\{3(4 \cdot 9 - 17 \cdot 2 + 2) - 3 \cdot 2 + 1\} = 2\{3(36 - 34 + 2) - 3 \cdot 2 + 1\} \\ &= 2\{3(2 + 2) - 3 \cdot 2 + 1\} = 2\{3 \cdot 4 - 3 \cdot 2 + 1\} = 2(12 - 6 + 1) = 2(6 + 1) = 2 \cdot 7 = 14 = \text{RHS } \checkmark \end{aligned}$$

So our solution, $\boxed{x = 2}$ is correct.

8. Solve each of the following application problems.

a) Ann and Betty dine together. The total bill is \$38. Ann paid \$2 more than Betty. How much did Betty pay?

Solution: Let us denote the amount paid by Betty paid by x . Then Ann paid $x + 2$. The equation expresses the total amount paid:

$$\begin{array}{rcll} x + x + 2 & = & 38 & \text{combine like terms} \\ 2x + 2 & = & 38 & \text{subtract } 2 \\ 2x & = & 36 & \text{divide by } 2 \\ x & = & 18 & \end{array}$$

Thus $\boxed{\text{Betty paid \$18 and Ann paid \$20.}}$

b) 55 people showed up on the party. There were 3 less women than men. How many men were there?

Solution: Let us denote the number of women by x . Then $x + 3$ men showed up. The equation expresses the total number of people:

$$x + x + 3 = 55 \quad \text{combine like terms}$$

$$2x + 3 = 55 \quad \text{subtract 3}$$

$$2x = 52 \quad \text{divide by 2}$$

$$x = 26$$

Thus there were 26 women and 29 men on the party.

c) One side of a rectangle is 7 cm shorter than five times the other side. Find the length of the sides if the perimeter of the rectangle is 118 cm.

Solution: Let us denote the shorter side by x . Then the longer side is $5x - 7$. We obtain the equation for the perimeter:

$$2x + 2(5x - 7) = 118 \quad \text{distribute}$$

$$2x + 10x - 14 = 118 \quad \text{combine like terms}$$

$$12x - 14 = 118 \quad \text{add 14}$$

$$12x = 132 \quad \text{divide by 12}$$

$$x = 11$$

Thus the shorter side is 11 cm, the longer side is $5(11 \text{ cm}) - 7 \text{ cm} = 48 \text{ cm}$. We check: the perimeter is $2(11 \text{ cm}) + 2(48 \text{ cm}) = 118 \text{ cm}$ and 48 is indeed 7 shorter than five times 11. Thus the solution is: 11 cm by 48 cm.

d) One side of a rectangle is 3 cm longer than four times the other side. Find the sides if the perimeter of the rectangle is 216 cm.

Solution: Let us denote the shorter side by x . Then the longer side is $4x + 3$. We obtain the equation for the perimeter:

$$2x + 2(4x + 3) = 216 \quad \text{distribute}$$

$$2x + 8x + 6 = 216 \quad \text{combine like terms}$$

$$10x + 6 = 216 \quad \text{subtract 6}$$

$$10x = 210 \quad \text{divide by 10}$$

$$x = 21$$

Thus the shorter side is 21 cm, the longer side is $4 \cdot 21 + 3 = 87 \text{ cm}$. We check: the perimeter is $2 \cdot 21 + 2 \cdot 87 = 42 + 174 = 216 \text{ cm}$ and 87 is indeed 3 longer than four times 21. Thus the solution is: 21 cm by 87 cm.

e) The sum of two consecutive even integers is -170 . Find these numbers.

Solution: Let us denote the smaller number by x . Then the larger number is $x + 2$. The equation expresses the sum of the numbers.

$$x + x + 2 = -170 \quad \text{combine like terms}$$

$$2x + 2 = -170 \quad \text{subtract 2}$$

$$2x = -172 \quad \text{divide by 2}$$

$$x = -86$$

Then the larger number must be $-86 + 2 = -84$. Thus the numbers are -86 and -84 .

f) The sum of three consecutive odd integers is 57. Find these numbers.

Solution: Let us denote the smallest number by x . Then the other two numbers must be $x + 2$ and $x + 4$. The equation expresses the sum of the three numbers.

$$\begin{aligned} x + x + 2 + x + 4 &= 57 && \text{combine like terms} \\ 3x + 6 &= 57 && \text{subtract 6} \\ x &= 51 && \text{divide by 3} \\ x &= 17 \end{aligned}$$

Thus the three numbers are 17, and $17 + 2 = 19$, and $17 + 4 = 21$. We check: indeed, $17 + 19 + 21 = 57$. Thus the solution is 17, 19, and 21.

g) Small ones weigh 3 lb, big ones weigh 4 lb. The number of small ones is 3 more than twice the number of big ones. All together, they weigh 79 lb. How many small ones are there?

Solution: Let us denote the number of big ones by x . Then the number of small ones is $2x + 3$. We obtain the equation expressing the total weight:

$$\begin{aligned} 3(2x + 3) + 4x &= 79 && \text{distribute} \\ 6x + 9 + 4x &= 79 && \text{combine like terms} \\ 10x + 9 &= 79 && \text{subtract 9} \\ 10x &= 70 && \text{divide by 10} \\ x &= 7 \end{aligned}$$

The number of big ones is then 7, and so the number of small ones is $2(7) + 3 = 17$. We check: the number of small ones, 17 is indeed 3 more than twice the number of big ones, 7. The total weight is $7(4) + 17(3) = 28 + 51 = 79$. Thus the solution is 7 big, 17 small.

h) We have a jar of coins, all quarters and dimes. All together, they are worth \$17.60 We have 13 more quarters than dimes. How many quarters, how many dimes?

Solution: Let us denote the number of dimes by x . Then the number of quarters must be $x + 13$. We obtain the equation by expressing the total value, **in pennies**.

$$\begin{aligned} 10x + 25(x + 13) &= 1760 && \text{distribute} \\ 10x + 25x + 325 &= 1760 && \text{combine like terms} \\ 35x + 325 &= 1760 && \text{subtract 325} \\ 35x &= 1435 && \text{divide by 35} \\ x &= 41 \end{aligned}$$

Thus we have 41 dimes and $41 + 13 = 54$ quarters. We check: $41(0.10) + 54(0.25) = 4.10 + 13.50 = 17.60$. Thus the solution is 41 dimes and 54 quarters.

i) Red pens cost \$2 each, blue ones cost \$3 each. We bought some pens. The number of red pens is 7 less than five times the number of blue pens. How many of each did we buy if we paid \$116?

Solution: Let us denote the number of blue pens by x . Then the number of red pens is $5x - 7$. The equation will express the total cost of the pens:

$$\begin{aligned}2(5x - 7) + 3(x) &= 116 && \text{distribute} \\10x - 14 + 3x &= 116 && \text{combine like terms} \\13x - 14 &= 116 && \text{add 14} \\13x &= 130 && \text{divide by 13} \\x &= 10\end{aligned}$$

Thus we bought 10 blue and $5(10) - 7 = 43$ red pens. We check:

$$\begin{aligned}43 &= 5(10) - 7 \checkmark \\2(43) + 3(10) &= 86 + 30 = 116 \checkmark\end{aligned}$$

Thus our solution is correct; we bought 10 blue and 43 red pens.

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