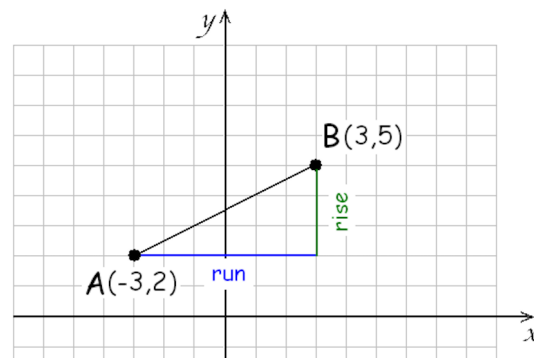


Definition: Equations that are in x , or in y , or in x and y can be graphed. **The graph of such an equation is the set of all points $P(x, y)$ for which the coordinates x and y form a solution of the equation.**

If the equation is linear in x and y , the graph of the equation is a line. If one of the variables is missing, and the equation is linear in the other (such as $x = 2$ or $y = -3$), the graph of the equation is a line.

The slope of a line is a very important concept. It expresses the steepness of a line. Given two points on a line, the slope is a signed ratio expressing the vertical change per the horizontal change between the two points.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in the } y\text{-coordinate}}{\text{change in the } x\text{-coordinate}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Definition: The slope of a line (usually denoted by m) is a signed ratio expressing the steepness of the line. Given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the slope of the line connecting these points is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

This formula is called the **slope formula**.

Example 1. In each case, find the slope of the line determined by the two points given.

- a) $(5, -1)$ and $(1, 3)$ b) $(2, 1)$ and $(6, 3)$ c) $(8, -1)$ and $(-1, -1)$ d) $(7, 3)$ and $(7, -4)$

Solution: a) We find the slope determined by the points. $(5, -1) = (x_1, y_1)$ and $(1, 3) = (x_2, y_2)$ using the slope formula.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-1)}{1 - 5} = \frac{4}{-4} = -1$$

So the slope is -1 .

- b) We find the slope determined by the points. $(2, 1) = (x_1, y_1)$ and $(6, 3) = (x_2, y_2)$ using the slope formula.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{6 - 2} = \frac{2}{4} = \frac{1}{2}$$

So the slope is $\frac{1}{2}$.

- c) We find the slope determined by the points. $(8, -1) = (x_1, y_1)$ and $(-1, -1) = (x_2, y_2)$ using the slope formula.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-1)}{-1 - 8} = \frac{0}{-9} = 0$$

So the slope is 0. Note that **the slope of all horizontal lines is zero**.

- d) We find the slope determined by the points. $(7, 3) = (x_1, y_1)$ and $(7, -4) = (x_2, y_2)$ using the slope formula.

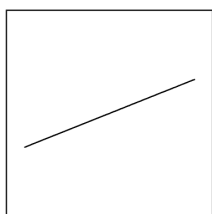
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 3}{7 - 7} = \frac{-7}{0} = \text{undefined}$$

So this line has no slope. Note that **vertical lines have no slope**.

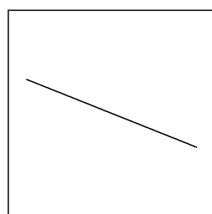


Discussion: Suppose we are finding the slope of a line connecting $A(-5, 2)$ and $B(1, 10)$. Ann is using point A as (x_1, y_1) and point B as (x_2, y_2) . Beth is using point B as (x_1, y_1) and point A as (x_2, y_2) . Compute the slope both ways. Do we get the same answer? Why or why not?

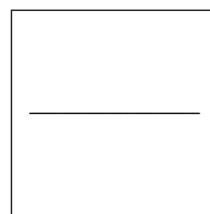
Note that **the slope of all horizontal lines is zero** and that **vertical lines have no slope**.



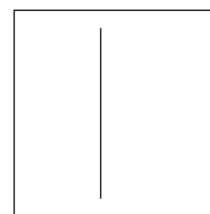
line with positive slope



line with negative slope



line with slope $m = 0$



line with no slope

When the equation of a line is given, we can find its slope by finding two points on the line and applying the slope formula. We will in the future develop other ways to find the slope of a line, given its equation.

Example 2. Find the slope of the line $2x - 3y = -12$.

Solution: We will find two points on the line and then use the slope formula to compute the slope. We can use any two points, the slope will always be the same. We will use $x = 0$ and $x = 6$. We will substitute these values into x in the equation of the line and solve for y .

If $x = 0$, then $2x - 3y = -12$ becomes

$$-3y = -12 \quad \text{divide by } -3$$

$$y = 4$$

and so $A(0, 4)$ is a point on our line.

If $x = 6$, then $2x - 3y = -12$ becomes

$$12 - 3y = -12 \quad \text{subtract } 12$$

$$-3y = -24 \quad \text{divide by } -3$$

$$y = 8$$

and so $B(6, 8)$ is a point on our line.

We are now ready to find the slope using the slope formula.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{6 - 0} = \frac{4}{6} = \frac{2}{3}$$

Example 3. Consider the line determined by the points $A(-4, 1)$ and $B(3, k)$. Find the value of k if the slope is 2.

Solution: We will state the slope formula for the line and then solve the resulting equation for k .

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{becomes} \quad 2 = \frac{k - 1}{3 - (-4)}$$

We simplify the denominator and solve for k .

$$\begin{array}{ll} 2 = \frac{k-1}{7} & \text{multiply by 7} \\ 14 = k-1 & \text{add 1} \\ 15 = k & \end{array}$$



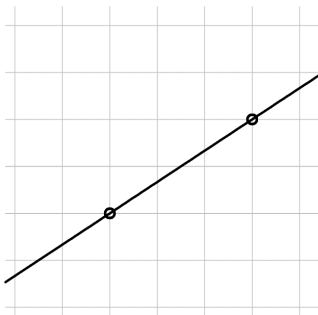
Practice Problems

1. In each case, determine the slope of the line connecting the points given.

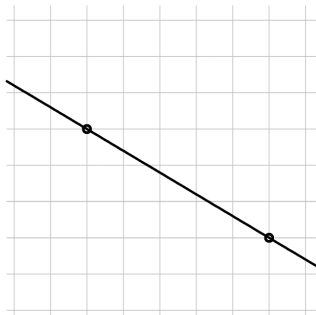
- a) $(-2, 7)$ and $(3, -3)$ b) $(7, -5)$ and $(3, -8)$ c) $(3, -2)$ and $(3, 6)$ d) $(2, 7)$ and $(10, 7)$

2. Determine the slope of each of the following lines, based on their graphs.

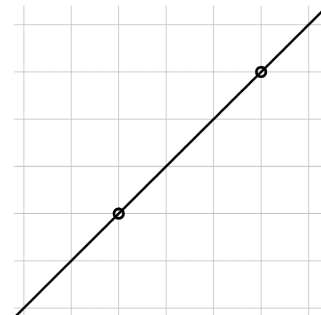
a)



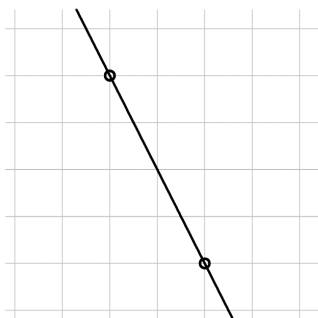
b)



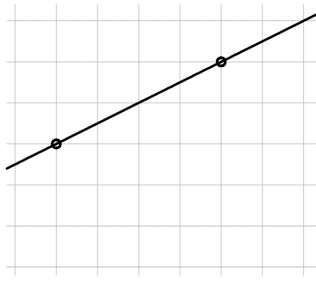
c)



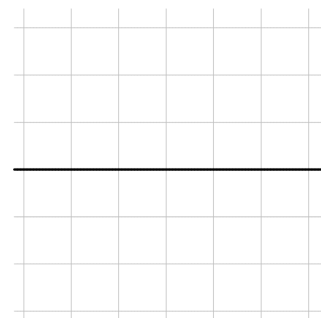
d)



e)



f)



3. Consider the line determined by the points $P(-2, r)$ and $Q(1, 4)$. Find the value of r if the slope of the line is

- a) 2 b) -1 c) 0 d) $-\frac{2}{3}$



Answers

1. a) -2 b) $\frac{3}{4}$ c) undefined d) 0

2. a) $m = \frac{2}{3}$ b) $m = -\frac{3}{5}$ c) $m = 1$ d) $m = -2$ e) $m = \frac{1}{2}$ f) $m = 0$

3. a) -2 b) 7 c) 4 d) 6