

Sample Problems

1. Suppose that r and s are real numbers $r + s = 3$ and $rs = -8$. Find the exact value (without computing r and s) of each of the following.

a) $r^2 + s^2$ b) $\frac{1}{r} + \frac{1}{s}$ c) $(r - s)^2$ d) $r^3 + s^3$ e) $r^4 + s^4$

2. (From Christian Sanchez) Let m and n be real numbers such that $m^2 + n^2 = 4$ and $m^4 + n^4 = 10$. Compute the exact value of each of the following

a) $(nm)^2$ b) $m^6 + n^6$ c) $m^{12} + n^{12}$ c) $(m^6 - n^6)^2$

Practice Problems

1. Let x and y be real numbers such that $x + y = -9$ and $xy = 7$. Without computing the values of x and y , compute the exact value of each of the following.

a) $x^2 + y^2$ b) $\frac{1}{x^2} + \frac{1}{y^2}$ c) $(x - y)^2$ d) $x^3 + y^3$ e) $x^4 + y^4$

2. Let p and q be real numbers such that $p + q = 1$ and $pq = -18$. Without computing the values of p and q , compute the exact value of each of the following.

a) $p^2 + q^2$ b) $\frac{2}{\frac{1}{p} + \frac{1}{q}}$ c) $\frac{p}{q} + \frac{q}{p}$ d) $p^3 + q^3$ e) $p^4 + q^4$

Sample Problems - Answers

$$1. \quad \text{a) } 25 \quad \text{b) } -\frac{3}{8} \quad \text{c) } 41 \quad \text{d) } 99 \quad \text{e) } 497$$

$$2. \quad \text{a) } 3 \quad \text{b) } 28 \quad \text{c) } 730 \quad \text{d) } 676$$

Practice Problems - Answers

$$1. \quad \text{a) } 67 \quad \text{b) } \frac{67}{49} \quad \text{c) } 53 \quad \text{d) } -540 \quad \text{e) } 4391$$

$$2. \quad \text{a) } 37 \quad \text{b) } -36 \quad \text{c) } -\frac{37}{18} \quad \text{d) } 55 \quad \text{e) } 721$$

Sample Problems - Solutions

1. Suppose that r and s are real numbers $r + s = 3$ and $rs = -8$. Find the exact value (without computing r and s) of each of the following.

$$\text{a) } r^2 + s^2 =$$

Solution: We start by squaring both sides of $r + s = 3$.

$$\begin{aligned} (r + s)^2 &= 9 \\ r^2 + s^2 + 2rs &= 9 \\ r^2 + s^2 + 2(-8) &= 9 \\ r^2 + s^2 - 16 &= 25 \end{aligned}$$

$$\text{b) } \frac{1}{r} + \frac{1}{s} =$$

$$\text{Solution: } \frac{1}{r} + \frac{1}{s} = \frac{s + r}{rs} = \frac{3}{-8} = -\frac{3}{8}$$

$$\text{b) } (r - s)^2 =$$

Solution: Note that we have already computed the value of $r^2 + s^2$.

$$(r - s)^2 = \underbrace{r^2 + s^2}_{25} - 2\underbrace{(rs)}_{-8} = 25 - 2(-8) = 41$$

c) $r^3 + s^3 =$

Solution: We start by raising both sides of $r + s = 3$ to the third power.

$$\begin{aligned} (r + s)^3 &= 3^3 & r^3 + s^3 + 3(-8)(3) &= 27 \\ r^3 + 3r^2s + 3rs^2 + s^3 &= 27 & r^3 + s^3 - 72 &= 27 \\ r^3 + s^3 + 3r^2s + 3rs^2 &= 27 & r^3 + s^3 &= 99 \\ r^3 + s^3 + 3rs(r + s) &= 27 \end{aligned}$$

d) $r^4 + s^4 =$

Solution: We have already computed $r^2 + s^2$. We start by squaring that quantity.

$$\begin{aligned} r^2 + s^2 &= 25 \\ (r^2 + s^2)^2 &= (25)^2 \\ r^4 + s^4 + 2r^2s^2 &= 625 \\ r^4 + s^4 + 2(rs)^2 &= 625 \\ r^4 + s^4 + 2(-8)^2 &= 625 \\ r^4 + s^4 + 128 &= 625 \\ r^4 + s^4 &= 497 \end{aligned}$$

2. (From Christian Sanchez) Let m and n be real numbers such that $m^2 + n^2 = 4$ and $m^4 + n^4 = 10$. Compute the exact value of each of the following

a) $(nm)^2$ b) $m^6 + n^6$ c) $m^{12} + n^{12}$ d) $(m^6 - n^6)^2$

Solution:

$$\begin{aligned} \text{a) } \quad (m^2 + n^2)^2 &= 4^2 \\ m^4 + n^4 + 2n^2m^2 &= 16 \\ 10 + 2n^2m^2 &= 16 \\ 2n^2m^2 &= 6 \\ (nm)^2 &= 3 \\ \text{b) } \quad (m^2 + n^2)(m^4 + n^4) &= 4 \cdot 10 \\ m^6 + m^4n^2 + m^2n^4 + n^6 &= 40 \\ m^6 + n^6 + m^4n^2 + m^2n^4 &= 40 \\ m^6 + n^6 + m^2n^2(m^2 + n^2) &= 40 \\ m^6 + n^6 + (3)(4) &= 40 \\ m^6 + n^6 &= 28 \\ \text{c) } \quad m^6 + n^6 &= 28 \\ (m^6 + n^6)^2 &= 784 \\ m^{12} + n^{12} + 2m^6n^6 &= 784 \\ m^{12} + n^{12} + 2((mn)^2)^3 &= 784 \\ m^{12} + n^{12} + 2(3)^3 &= 784 \\ m^{12} + n^{12} + 54 &= 784 \\ m^{12} + n^{12} &= 730 \\ \text{d) } \quad (m^6 - n^6)^2 &= m^{12} + n^{12} - 2m^6n^6 \\ &= m^{12} + n^{12} - 2m^6n^6 \\ &= 730 - 2((mn)^2)^3 \\ &= 730 - 2 \cdot 3^3 \\ &= 676 \end{aligned}$$

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