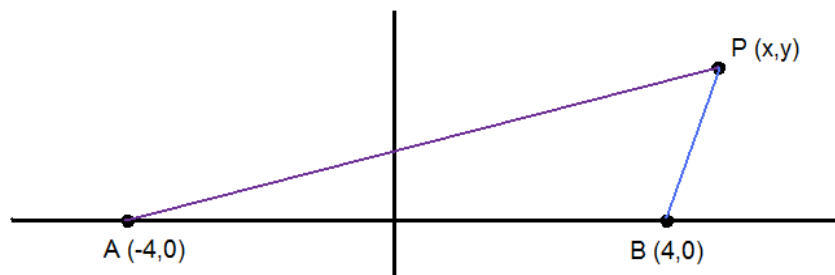


Notation: Let the distance between points A and B be denoted by $d(A, B)$

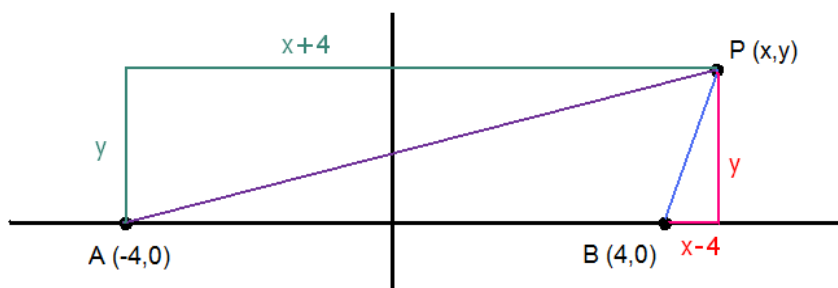
Let $A(-4, 0)$ and $B(4, 0)$ be given. Find the equation for the set of all points $P(x, y)$ such that $d(P, A) + d(P, B) = 10$.



We can express these distances as

$$d(P, A) = \sqrt{(x+4)^2 + y^2} \quad \text{and} \quad d(P, B) = \sqrt{(x-4)^2 + y^2}$$

based on the figure shown below.



Using our expressions for $d(P, A)$ and $d(P, B)$ in terms of x and y , $d(P, A) + d(P, B) = 10$ means that

$$\begin{aligned} \sqrt{(x+4)^2 + y^2} + \sqrt{(x-4)^2 + y^2} &= 10 \\ \sqrt{(x+4)^2 + y^2} &= 10 - \sqrt{(x-4)^2 + y^2} \end{aligned}$$

We square both sides:

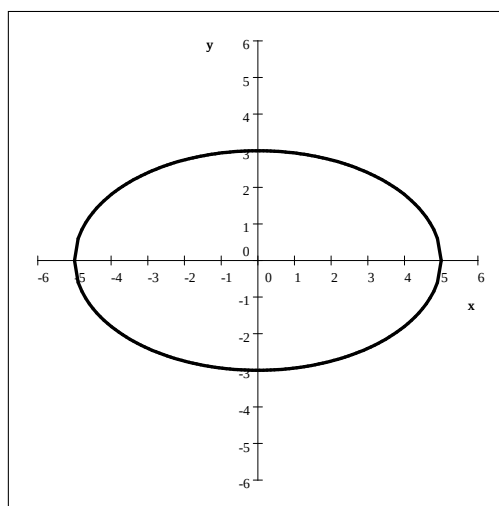
$$\begin{aligned} (x+4)^2 + y^2 &= \left(10 - \sqrt{(x-4)^2 + y^2}\right)^2 \\ (x+4)^2 + y^2 &= 100 + (x-4)^2 + y^2 - 20\sqrt{(x-4)^2 + y^2} \\ (x+4)^2 &= 100 + (x-4)^2 - 20\sqrt{(x-4)^2 + y^2} \\ 20\sqrt{(x-4)^2 + y^2} &= 100 + (x-4)^2 - (x+4)^2 \end{aligned}$$

$$\begin{aligned}
20\sqrt{(x-4)^2 + y^2} &= 100 + (x^2 - 8x + 16) - (x^2 + 8x + 16) \\
20\sqrt{(x-4)^2 + y^2} &= 100 + x^2 - 8x + 16 - x^2 - 8x - 16 \\
20\sqrt{(x-4)^2 + y^2} &= 100 - 16x \quad \text{divide by 4} \\
5\sqrt{(x-4)^2 + y^2} &= 25 - 4x
\end{aligned}$$

We square both sides:

$$\begin{aligned}
25((x-4)^2 + y^2) &= (25 - 4x)^2 \\
25(x^2 - 8x + 16 + y^2) &= 625 - 200x + 16x^2 \\
25x^2 - 200x + 400 + 25y^2 &= 625 - 200x + 16x^2 \\
9x^2 + 25y^2 &= 225 \quad \text{divide by 225} \\
\frac{x^2}{25} + \frac{y^2}{9} &= 1
\end{aligned}$$

The graph of this ellipse is



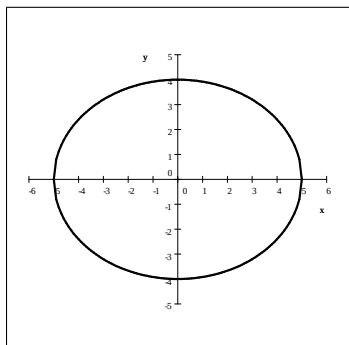
Practice Problems

Derive the formula for each of the following. Then graph the given set.

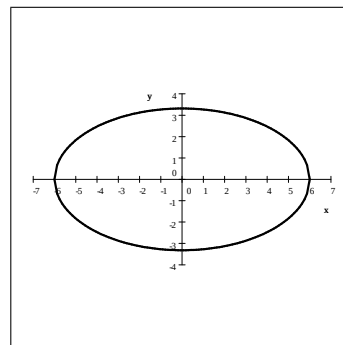
1. Let $A(3, 0)$ and $B(-3, 0)$. Consider the set of all points $P(x, y)$ so that $d(P, A) + d(P, B) = 10$.
2. Let $A(0, 2)$ and $B(0, -2)$. Consider the set of all points $P(x, y)$ so that $d(P, A) + d(P, B) = 6$.
3. Let $A(5, 0)$ and $B(-5, 0)$. Consider the set of all points $P(x, y)$ so that $d(P, A) + d(P, B) = 12$.
4. Let $A(\sqrt{2}, 0)$ and $B(-\sqrt{2}, 0)$. Consider the set of all points $P(x, y)$ so that $d(P, A) + d(P, B) = 4$.

Answers - Practice Problems

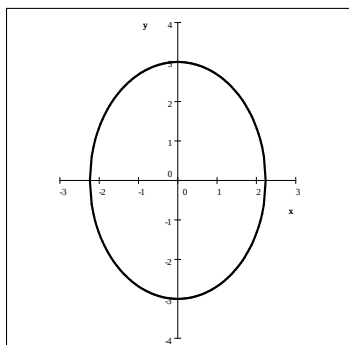
1. $\frac{x^2}{25} + \frac{y^2}{16} = 1$



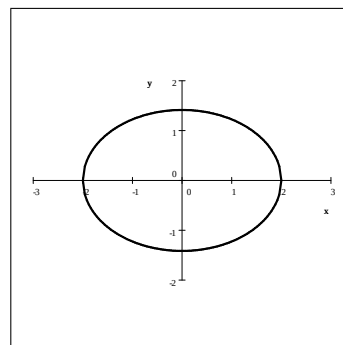
3. $\frac{x^2}{36} + \frac{y^2}{11} = 1$



2. $\frac{x^2}{5} + \frac{y^2}{9} = 1$



4. $\frac{x^2}{4} + \frac{y^2}{2} = 1$



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