

Completely factor each of the following polynomials over the complex numbers and over the real numbers.

1. $Q(x) = x^2 + 1$
2. $Q(x) = x^4 - 16$
3. $P(x) = x^3 + x^2 + 20x + 78$ and $P(-3) = 0$
4. $P(x) = x^3 - 13x^2 + 67x - 175$ and $P(7) = 0$
5. $Q(x) = x^5 - 10x^4 - 2x^3 - 90x^2 - 99x$, and $Q(3i) = 0$
6. $Q(x) = x^6 - 2x^5 + 11x^4 - 2x^3 + 10x^2$ and $Q(-i) = 0$
7. $P(x) = x^4 + 12x^3 + 37x^2 + 40x$ and $P(-2 - i) = 0$
8. $Q(x) = x^2 - 1$
9. $Q(x) = x^3 - 1$
10. $Q(x) = x^4 - 1$

Answers

1. over $\mathbb{C}$: $(x + i)(x - i)$
over $\mathbb{R}$: $x^2 + 1$
2. over $\mathbb{C}$: $(x + 2i)(x - 2i)(x^2 - 4)$
over $\mathbb{R}$: $(x^2 + 4)(x + 2)(x - 2)$
3. over $\mathbb{C}$: $(x + 3)(x - (1 + 5i))(x - (1 - 5i))$
over $\mathbb{R}$: $(x + 3)(x^2 - 2x + 26)$
4. over $\mathbb{C}$: $(x - 7)(x - (3 + 4i))(x - (3 - 4i))$
over $\mathbb{R}$: $(x - 7)(x^2 - 6x + 25)$
5. over $\mathbb{C}$: $(x + 3i)(x - 3i)x(x + 1)(x - 11)$
over $\mathbb{R}$: $x(x^2 + 9)(x + 1)(x - 11)$
6. over $\mathbb{C}$: $(x + i)(x - i)x^2(x - (1 + 3i))(x - (1 - 3i))$
over $\mathbb{R}$: $x^2(x^2 + 1)(x^2 - 2x + 10)$
7. over $\mathbb{C}$: $(x + 8)x(x - (-2 + i))(x - (-2 - i))$
over $\mathbb{R}$: $x(x + 8)(x^2 + 4x + 5)$
8. over $\mathbb{C}$: $(x - 1)(x + 1)$
over $\mathbb{R}$: $(x - 1)(x + 1)$
9. over $\mathbb{C}$: $(x - 1)\left(x - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right)\left(x - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right)$
over $\mathbb{R}$: $(x - 1)(x^2 + x + 1)$
10. over $\mathbb{C}$: $(x - 1)(x + 1)(x + i)(x - i)$
over $\mathbb{R}$: $(x - 1)(x + 1)(x^2 + 1)$