

Sample Problems

Solve each of the following equations.

$$1.) \sin x = -\frac{\sqrt{3}}{2} \quad 2.) \cos x = -\frac{1}{2} \quad 3.) \tan x = -1 \quad 4.) \cos x = 2 \quad 5.) \sin x = -1$$

Practice Problems

1. Solve each of the following equations.

$$a) \tan x = -\frac{1}{\sqrt{3}} \quad d) \sin x = -\frac{1}{2} \quad c) \cos x = 1$$

$$b) \sin x = 0 \quad e) \sin x = -1.3 \quad f) \cos x = \frac{\sqrt{3}}{2}$$

2. Find the domain of each of the following functions.

$$a) f(x) = \tan x \quad b) g(x) = \cot x \quad c) t(x) = \frac{1}{\sin^2 x + \cos^2 x}$$

Sample Problems- Answers

- 1.) $-\frac{\pi}{3} + 2k\pi$ and $-\frac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ 2.) $x = \pm \frac{2\pi}{3} + 2k\pi$ $k \in \mathbb{Z}$
 3.) $x = -\frac{\pi}{4} + k\pi$, where $k \in \mathbb{Z}$ 4.) no solution 5.) $-\frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$

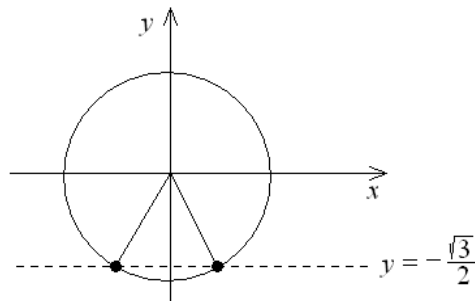
Practice Problems - Answers

1. a) $-\frac{1}{6}\pi + k\pi$ $k \in \mathbb{Z}$ b) $k\pi$ $k \in \mathbb{Z}$ c) $2k\pi$ $k \in \mathbb{Z}$
 d) $-\frac{1}{6}\pi + 2k_1\pi$ and $\frac{7}{6}\pi + 2k_2\pi$ $k_1, k_2 \in \mathbb{Z}$ e) no solution f) $\pm \frac{\pi}{6} + 2k\pi$ $k \in \mathbb{Z}$
 2. a) $x \neq \frac{\pi}{2} + k\pi$ $k \in \mathbb{Z}$ b) $x \neq k\pi$ $k \in \mathbb{Z}$ c) $\mathbb{R}$

Sample Problems - Solutions

- 1.) Solve the equation $\sin x = -\frac{\sqrt{3}}{2}$.

Step 1. We draw a unit circle to find all solutions on it first. $\sin x = -\frac{\sqrt{3}}{2}$ means that we are looking for all points on it with second coordinate $-\frac{\sqrt{3}}{2}$. The two solutions on the unit circle are the intersections of the unit circle and the line $y = -\frac{\sqrt{3}}{2}$.



Step 2. We translate the solutions to two angles. In this case, -60° and -120° are two reasonable choices. (240° and 300° are also correct).

Step 3. We algebraically state the result, which are the previously found two angles, and all co-terminal angles.

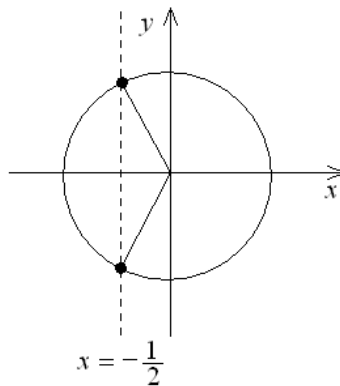
$$x = -60^\circ + k \cdot 360^\circ \text{ or } x = -120^\circ + k \cdot 360^\circ \text{ where } k = 0, 1, -1, 2, -2, 3, -3, \dots$$

Step 4. Convert the solutions to radians. Remember, $180^\circ = \pi$ radians.

$$x = -\frac{\pi}{3} + 2k\pi \text{ or } x = -\frac{2\pi}{3} + 2k\pi \text{ where } k = 0, 1, -1, 2, -2, 3, -3, \dots$$

2.) Solve the equation $\cos x = -\frac{1}{2}$.

Step 1. We draw a unit circle to find all solutions on it first. $\cos x = -\frac{1}{2}$ means that we are looking for all points on it with first coordinate $-\frac{1}{2}$. The two solutions on the unit circle are the intersections of the unit circle and the line $x = -\frac{1}{2}$.



Step 2. We translate the solutions to two angles. In this case, 120° and -120° are two reasonable choices. (120° and 240° are also correct).

Step 3. We algebraically state the result, which are the previously found two angles, and all co-terminal angles.

$$x = 120^\circ + k \cdot 360^\circ \text{ or } x = -120^\circ + k \cdot 360^\circ \text{ where } k \in \mathbb{Z}$$

Notice that the new notation, $k \in \mathbb{Z}$ is a shorter way to write $k = 0, 1, -1, 2, -2, 3, -3, \dots$. In the same spirit, the result (unlike with $\sin x$) can be written as a single line, as

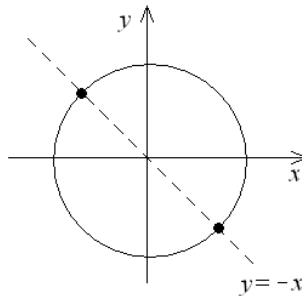
$$x = \pm 120^\circ + k \cdot 360^\circ \text{ where } k \in \mathbb{Z}$$

Step 4. Convert the solutions to radians. Remember, $180^\circ = \pi$ radians.

$$x = \pm \frac{2\pi}{3} + 2k\pi \quad k \in \mathbb{Z}$$

3.) Solve the equation $\tan x = -1$.

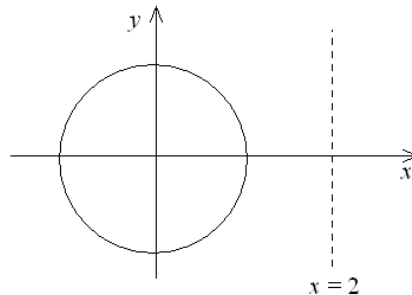
Like in the case of sine and cosine, there will also be two solutions on the unit circle. Also, they will be the intersections of a line and the unit circle. $\tan x = -1$ is true when $x = -45^\circ$. The second solution on the unit circle can be found by continuing the same straight line that connected -45° with the origin. The solutions are now on the line of whose **slope** is -1 .



The solutions are $x = -45^\circ + k \cdot 180^\circ$ where $k \in \mathbb{Z}$. Or, in radians, $x = -\frac{\pi}{4} + k\pi$, where $k \in \mathbb{Z}$.

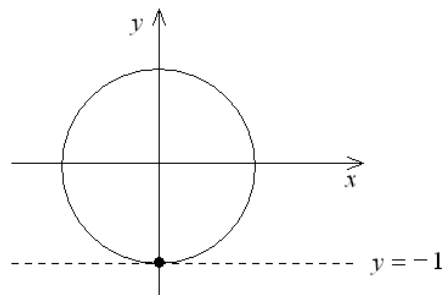
4.) Solve the equation $\cos x = 2$.

We draw a unit circle to find all solutions on it first. $\cos x = 2$ means that we are looking for all points on it with first coordinate 2. The solutions would be the intersections of the unit circle and the line $x = 2$. Since the circle and line do not intersect, there are no solutions to this equation.



5.) Solve the equation $\sin x = -1$.

We draw a unit circle to find all solutions on it first. $\sin x = -1$ means that we are looking for all points on it with second coordinate -1 . There is only one solution on the unit circle because the line $y = -1$ is a tangent line.



The intersection point corresponds to -90° (or 270°).

$$x = -90^\circ + k \cdot 360^\circ \quad k \in \mathbb{Z} \quad \text{or in radians:} \quad -\frac{\pi}{2} + 2k\pi \quad \text{where } k \in \mathbb{Z}$$

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