

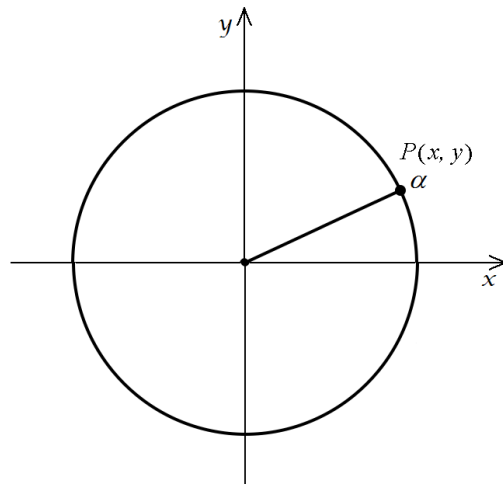
The Unit Circle Definition of Trigonometric Functions

Let α be **any** rotational angle. (It could be negative, or greater than 180° , or even greater than 360° .) We will define all six trigonometric functions of α as follows.

Step 1. Draw a unit circle and draw a ray representing α .

Step 2. For any angle α , this ray intersects the unit circle in a single point $P(x, y)$.

Step 3. We define the trigonometric function values of α as:



$$\cos \alpha = x$$

$$\sin \alpha = y$$

$$\tan \alpha = \frac{y}{x}$$

$$\sec \alpha = \frac{1}{x}$$

$$\csc \alpha = \frac{1}{y}$$

$$\cot \alpha = \frac{x}{y}$$

$\cos \alpha$ and $\sin \alpha$ always exists. For the other four trigonometric functions, we will need to worry about division by zero. Since $\tan \alpha = \frac{y}{x}$, it will be defined when $x \neq 0$. Similarly, $\sec \alpha = \frac{1}{x}$ is defined when $x \neq 0$ and $\csc \alpha = \frac{1}{y}$ and $\cot \alpha = \frac{x}{y}$ will be defined if $y \neq 0$.

Practice Problems

In case of each of the following angles, graph the angle in the unit circle and state its sine, cosine, and tangent.

1. 180°

3. 210°

5. 270°

7. 90°

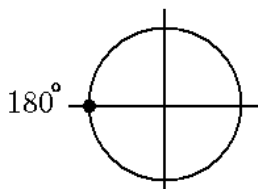
2. -60°

4. 135°

6. -135°

8. 150°

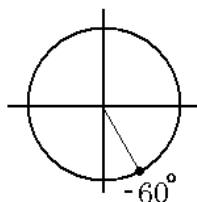
Answers - Practice Problems

1.) 180° 

$$\cos 180^\circ = -1$$

$$\sin 180^\circ = 0$$

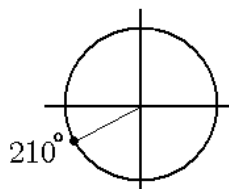
$$\tan 180^\circ = 0$$

2.) -60° 

$$\cos(-60^\circ) = \frac{1}{2}$$

$$\sin(-60^\circ) = -\frac{\sqrt{3}}{2}$$

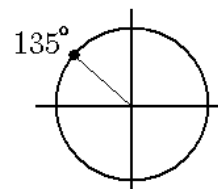
$$\tan(-60^\circ) = -\sqrt{3}$$

3.) 210° 

$$\cos 210^\circ = -\frac{\sqrt{3}}{2}$$

$$\sin 210^\circ = -\frac{1}{2}$$

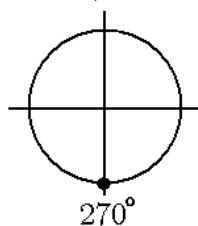
$$\tan 210^\circ = \frac{\sqrt{3}}{3}$$

4.) 135° 

$$\cos 135^\circ = -\frac{\sqrt{2}}{2}$$

$$\sin 135^\circ = \frac{\sqrt{2}}{2}$$

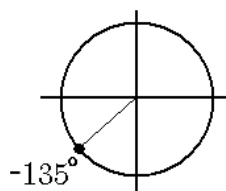
$$\tan 135^\circ = -1$$

5.) 270° 

$$\cos 270^\circ = 0$$

$$\sin 270^\circ = -1$$

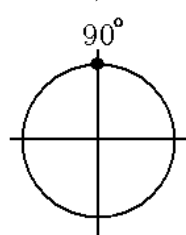
$$\tan 270^\circ = \text{undefined}$$

6.) -135° 

$$\cos(-135^\circ) = -\frac{\sqrt{2}}{2}$$

$$\sin(-135^\circ) = -\frac{\sqrt{2}}{2}$$

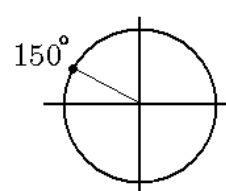
$$\tan(-135^\circ) = 1$$

7.) 90° 

$$\cos 90^\circ = 0$$

$$\sin 90^\circ = 1$$

$$\tan 90^\circ = \text{undefined}$$

8.) 150° 

$$\cos 150^\circ = -\frac{\sqrt{3}}{2}$$

$$\sin 150^\circ = \frac{1}{2}$$

$$\tan 150^\circ = -\frac{\sqrt{3}}{3}$$