

Calculus

Lectures

Average Speed and Velocity

Average Velocity

Complete Analysis of a Function - Part 1

Limits at Infinity - Part 1 and Part 2

Two-Sided Limits

The First Derivative Test

Proving the Intermediate Value Theorem

The Bolzano-Weierstrass Theorem

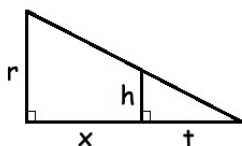
A Definition of e

Examples

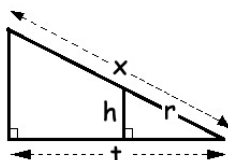
The problems here are not calculus problem, they are just part of the review at the beginning of the course.

1. Express x in terms of the other variables in the picture.

[Solution](#)

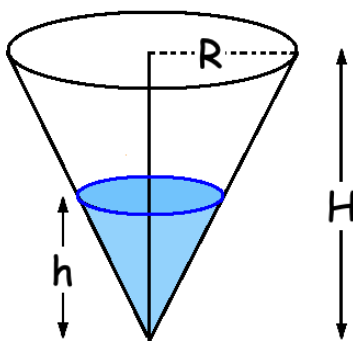


a)



b)

2. A water tank is of the shape of a cone. The tank is $H = 20\text{ft}$ tall and the circular top edge of the cone has a radius $R = 5\text{ft}$ long. If the surface of the water is at a height of $h = 3\text{ft}$, what is the surface area of the top of the water? [Solution](#)



3. Compute the given limits.

a) $\lim_{x \rightarrow \infty} \frac{2^x + 2^{-x}}{2^x - 2^{-x}}$

b) $\lim_{x \rightarrow -\infty} \frac{2^x + 2^{-x}}{2^x - 2^{-x}}$

[Solution](#)

4. Find all relative extrema for the function $f(x) = 2x^3 + 6x^2 - 54x - 15$. State the intervals over which f is increasing and decreasing. [Solution](#)

5. Find the slope of the tangent line drawn to the graph of $y = -\frac{1}{2}x^2 + 3x + 1$ from the point $P(4, 7)$. [Solution](#)

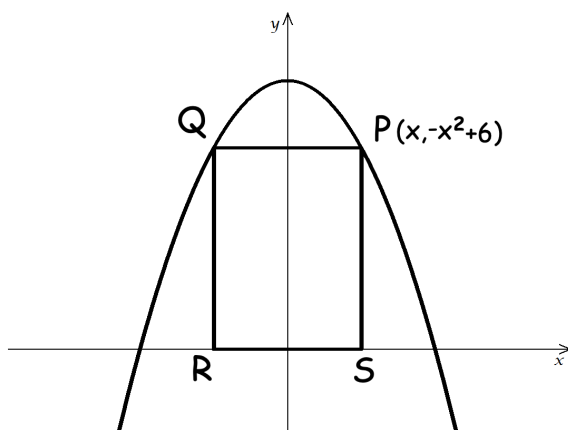
6. Find all values of b and c so that $y = 10x + 1$ is tangent to $f(x) = x^3 + bx^2 + cx - 3$. [Solution](#)

7. Find an equation of the common tangent lines drawn to the graphs of $y = x^2$ and $y = (x - 2)^2 + 12$. [Solution](#)

8. Find all values of p so that $y = 4x + 2$ is tangent to $y = px^2 + 8$. [Solution](#)

9. Find the greatest possible value of the product of two positive numbers if the sum of one number and the square of the other is 6. [Solution](#)

10. Let $P(x, y)$ be a point on the graph of $y = -x^2 + 6$ with $0 < x < \sqrt{6}$. Let PQRS be a rectangle with one side on the x-axis and two vertices on the graph, as shown on the picture. Find the exact value of the greatest possible area of such a rectangle. [Solution](#)



11. Find the point(s) on the graph of $y = 5x^2$ that is closest to the point $P(0, 2)$. [Solution](#)

12. Which square-based open box with a surface of 2400 square inches has the greatest volume? [Solution](#)

13. Compute the given limit. You may assume that $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$ and $\lim_{x \rightarrow -\infty} \left(1 + \frac{1}{x}\right)^x = e$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x \quad \text{Solution}$$

14. Compute each of the given indefinite integrals.

a) $\int e^{3x-2} dx$	e) $\int \cos^2 x dx$
b) $\int (5x + 8)^{10} dx$	f) $\int \frac{1}{3-x} dx$
c) $\int \cos(2x - \pi) dx$	g) $\int \frac{6x-1}{2x+5} dx$
d) $\int \sin x \cos x dx$	Solution